

1)  $f(a)$  is defined    2)  $\lim_{x \rightarrow a} f(x)$  exists    3)  $\lim_{x \rightarrow a} f(x) = f(a)$

Limits and Continuity  
Calculus Concepts  
Unit 1 - Worksheet 3

Name key  
Date

1. Let  $f(x) = \frac{x^2 - 9}{x + 3} = \frac{(x+3)(x-3)}{x+3}$

a.  $\lim_{x \rightarrow -3^-} f(x) = -6$

c.  $\lim_{x \rightarrow -3} f(x) = -6$

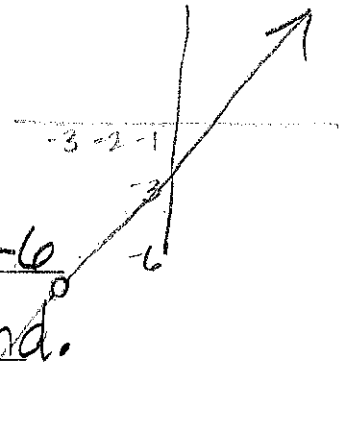
b.  $\lim_{x \rightarrow -3^+} f(x) = -6$

d.  $f(-3) = \text{und.}$

e. Is continuous at  $x = -3$ ? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

#1  
X

Not cont, hole @  $x = -3$ , factors cancel on top: bottom  
 $f(-3)$  is und.



2. Let  $f(x) = \begin{cases} 3x + 4 & x \leq -2 \\ x^2 + 1 & x > -2 \end{cases}$

a.  $\lim_{x \rightarrow -2^-} f(x) = -2$

c.  $\lim_{x \rightarrow -2} f(x) = \text{dne}$

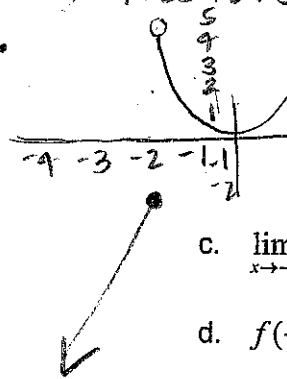
b.  $\lim_{x \rightarrow -2^+} f(x) = 5$

d.  $f(-2) = -2$

e. Is continuous at  $x = -2$ ? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

#2  
X

no,  $\lim_{x \rightarrow -2} \text{dne}$ , it's a jump, it's piecewise



3. Let  $f(x) = \begin{cases} \frac{x^2 + 3x + 2}{x + 2} & x \neq -2 \\ \frac{1}{2} & x = -2 \end{cases}$

a.  $\lim_{x \rightarrow -2^-} f(x) = -1$

c.  $\lim_{x \rightarrow -2} f(x) = -1$

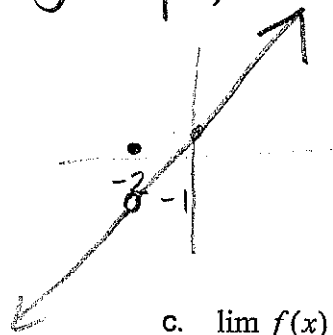
b.  $\lim_{x \rightarrow -2^+} f(x) = -1$

d.  $f(-2) = \frac{1}{2}$

e. Is continuous at  $x = -2$ ? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

#3  
X

no,  $\lim_{x \rightarrow -2} f(x) \neq f(-2)$ , a hole, factors cancel on top: bottom



4. Given  $f(x) = \begin{cases} 3x+2 & \text{if } x < 4 \\ 5x+k & \text{if } x \geq 4 \end{cases}$

Find the value of  $k$  such that  $\lim_{x \rightarrow 4} f(x)$  exists.

$$5x + k = 14$$
$$(4) \quad k = -6$$

5. Given  $f(x) = \begin{cases} 2x-a & \text{if } x < -3 \\ ax+2b & \text{if } -3 \leq x \leq 3 \\ b-5x & \text{if } x > 3 \end{cases}$

Find the values of  $a$  and  $b$  such that  $\lim_{x \rightarrow -3} f(x)$  and  $\lim_{x \rightarrow 3} f(x)$  both exist.

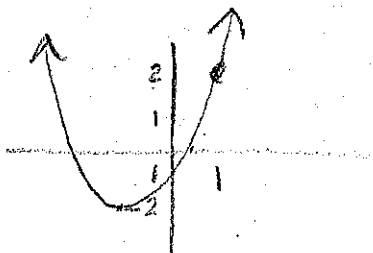
Please show all work.  
 Full credit will not be given if appropriate work is not shown.

Find each of the following limits. Please use proper notation.

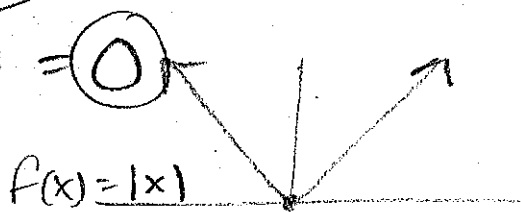
1.  $\lim_{x \rightarrow 1} (x^2 + 2x - 1)$  direct sub

$$1^2 + 2(1) - 1$$

$$1 + 2 - 1 = 3 - 1 = \textcircled{2}$$



2.  $\lim_{x \rightarrow 0} |x|$  direct sub  $|0| = \textcircled{0}$



3.  $\lim_{x \rightarrow 3} \left( \frac{x-3}{x^2-9} \right)$   $\frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3} = \frac{1}{3+3} = \textcircled{\frac{1}{6}}$

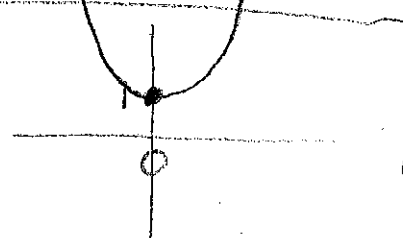
factor and cancel, then direct sub



4.  $\lim_{x \rightarrow 0} \left( \frac{1}{x} \right)$  Table and graph  
dne

V.A.  $x=0$

-0.0001	0	0.0001
-10,000	dne	10,000
$-\infty$		$+\infty$



factor top 5.  $f(x) = \begin{cases} \frac{x^3-1}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$  note @  $x=1$   $(x-1)(x^2+x+1)$   
 $\lim_{x \rightarrow 1} f(x) = 1$

\* When you sub/plug into both you get the same 'y' value  
This fills the hole