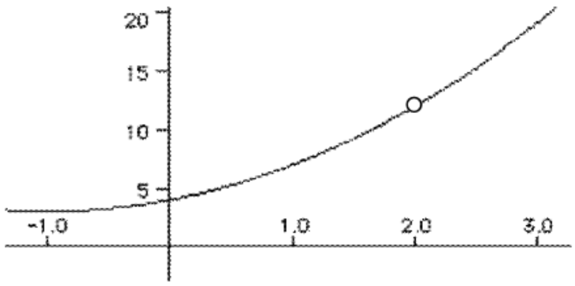
Graphical Approach to Limits - Classwork

Suppose you were to graph

$f(x) = \frac{x^3 - 8}{x - 2}, \quad x \neq 2$

For all values of x not equal to 2, you can use standard curve sketching techniques. But the curve is not defined at $x = 2$. There is a hole in the graph. So let's get an idea of the behavior of the curve around $x = 2$.



Set your calculator to 4 decimal accuracy and complete the chart.

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$f(x)$					und				

It should be obvious that as x gets closer and closer to 2, the value of $f(x)$ becomes closer and closer to 12.

We will say that the **limit** of $f(x)$ as x approaches 2 is 12 and this is written as $\lim_{x\to 2} f(x) = 12$ or $\lim_{x\to 2} \frac{x^3-8}{x-2} = 12$.

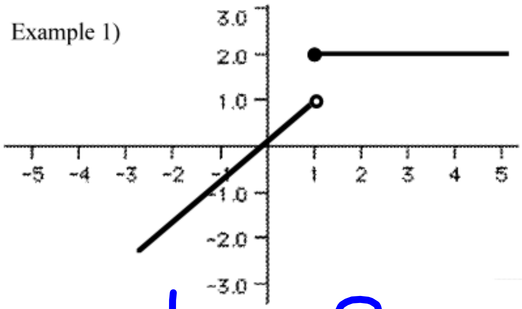
The informal of definition of a limit is "what is happening to y as x gets close to a certain number." In order for a limit to exist, we must be approaching the same y-value as we approach some value c from either the left or the right side. If this does not happen, we say that the limit does not exist (DNE) as we approach c.

If we want the limit of $f(x)$ as we approach some value of c from the left hand side, we will write $\lim_{x\to c^-} f(x)$.

If we want the limit of $f(x)$ as we approach some value of c from the right hand side, we will write $\lim_{x\to c^+} f(x)$.

In order for a limit to exist at c, $\lim_{x\to c^-} f(x)$ must equal $\lim_{x\to c^+} f(x)$ and we say $\lim_{x\to c} f(x) = L$.

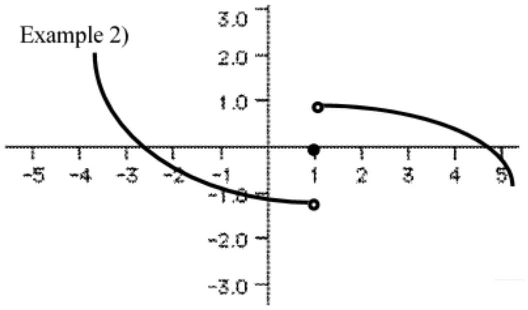
Example 1)



$\lim_{x\to 1^-} f(x) =$ 1 $\quad \lim_{x\to 1^+} f(x) =$ 2

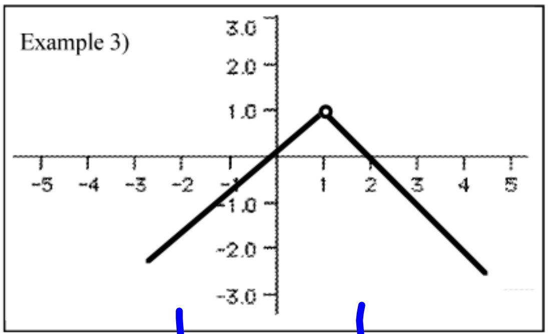
$\lim_{x\to 1} f(x) =$ DNE $\quad f(1) =$ 2

Example 2)



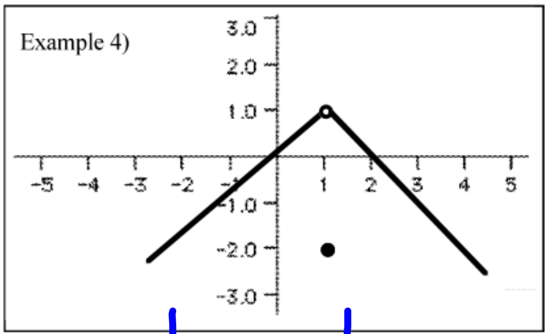
$\lim_{x\to 1^-} f(x) =$ -1 $\quad \lim_{x\to 1^+} f(x) =$ 1

$\lim_{x\to 1} f(x) =$ DNE $\quad f(1) =$ 0



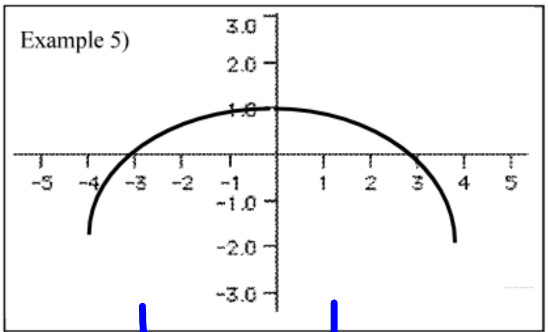
$\lim_{x\to 1^-} f(x) = 1$ $\lim_{x\to 1^+} f(x) = 1$

$\lim_{x\to 1} f(x) = 1$ $f(1) = \text{und}$



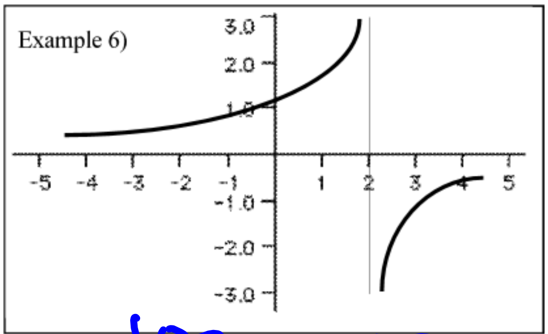
$\lim_{x\to 1^-} f(x) = 1$ $\lim_{x\to 1^+} f(x) = 1$

$\lim_{x\to 1} f(x) = 1$ $f(1) = -2$



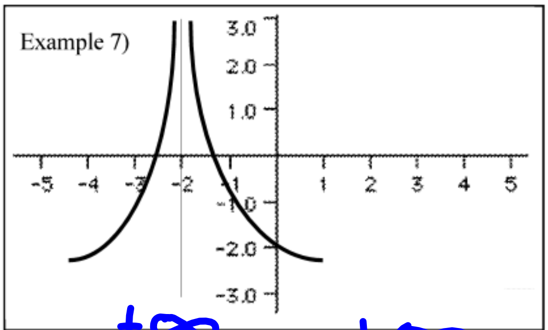
$\lim_{x\to 0^-} f(x) = 1$ $\lim_{x\to 0^+} f(x) = 1$

$\lim_{x\to 0} f(x) = 1$ $f(0) = 1$



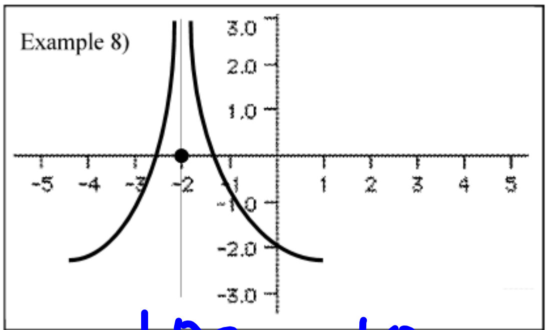
$\lim_{x\to 2^-} f(x) = +\infty$ $\lim_{x\to 2^+} f(x) = -\infty$

$\lim_{x\to 2} f(x) = \text{DNE}$ $f(2) = \text{und}$



$\lim_{x\to -2^-} f(x) = +\infty$ $\lim_{x\to -2^+} f(x) = +\infty$

$\lim_{x\to -2} f(x) = +\infty$ $f(-2) = \text{und}$



$\lim_{x\to -2^-} f(x) = +\infty$ $\lim_{x\to -2^+} f(x) = +\infty$

$\lim_{x\to -2} f(x) = +\infty$ $f(-2) = 0$

The concept of limits as x approaches infinity means the following: "what happens to y as x gets infinitely large." We are interested in what is happening to the y-value as the curve gets farther and farther to the right. We can also talk about limits as x approaches negative infinity. This means what is happening to the y-value as the curve gets farther and farther to the left. The terminology we use are the following: $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

Although we use the term "as x approaches infinity", realize that x cannot approach infinity as infinity does not exist. The term "x approaches infinity" is just a convenient way to talk about the curve infinitely far to the right.

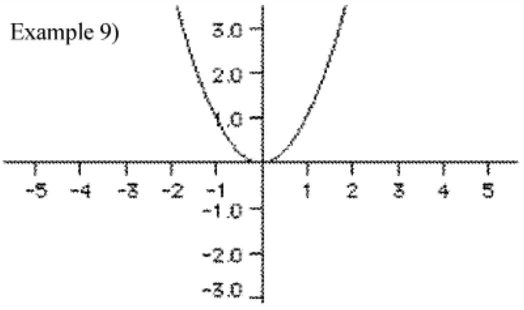
Note that it makes no sense to talk about $\lim_{x\to\infty^+} f(x)$ or $\lim_{x\to-\infty^-} f(x)$. Why? ______________________________

End Behavior

There are only 4 possibilities for $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$:

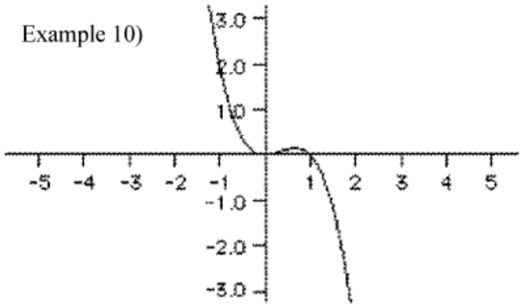
- the curve can go up forever. In that case, the limit does not exist. For convenience sake, we will say $\lim_{x\to\infty} f(x)=\infty$
- the curve can go down forever. In that case, the limit does not exist. For convenience sake we will say $\lim_{x\to\infty} f(x)=-\infty$

Example 9)



In this case, $\lim_{x\to-\infty} f(x) =$ $+\infty$

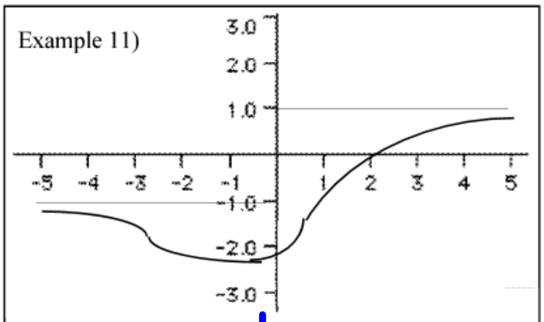
Example 10)



In this case, $\lim_{x\to-\infty} f(x) =$ $+\infty$

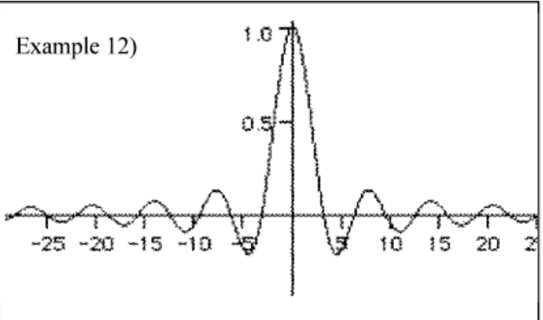
- the curve can become asymptotic to a line. In that case the limit as x approaches infinity is a value.
- the curve can level off to a line. In that case, the limit as x approaches infinity is a value.

Example 11)



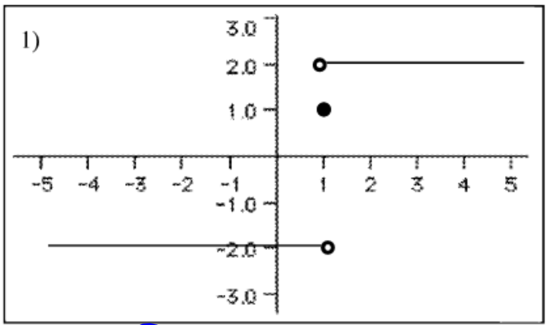
In this case, $\lim_{x\to\infty} f(x) =$ 1 and $\lim_{x\to-\infty} f(x) =$ -1

Example 12)

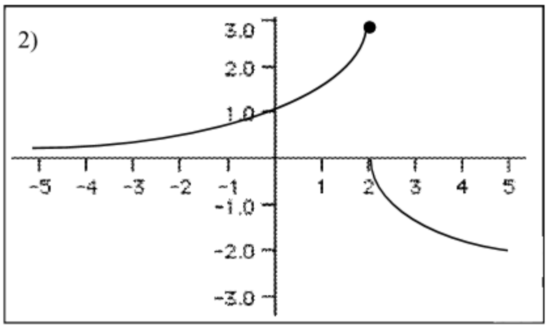


In this case, $\lim_{x\to\infty} f(x) =$ 0 and $\lim_{x\to-\infty} f(x) =$ 0

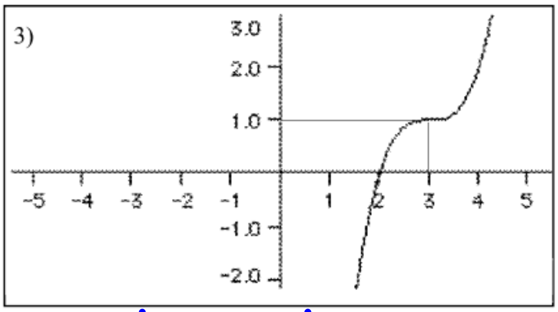
Graphical Approach to Limits - Homework

1) 

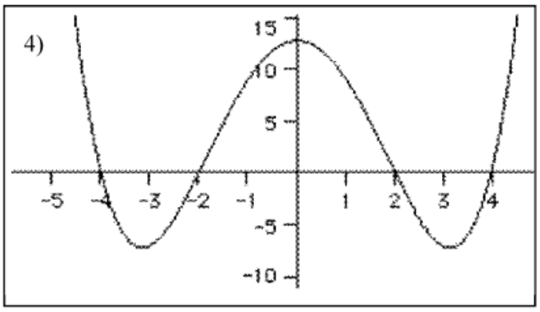
a) $\lim_{x\to1^-} f(x)$ -2 b) $\lim_{x\to1^+} f(x)$ 2 c) $\lim_{x\to1} f(x)$ DNE
d) $f(1)$ 1 e) $\lim_{x\to-\infty} f(x)$ -2 f) $\lim_{x\to\infty} f(x)$ 2

2) 

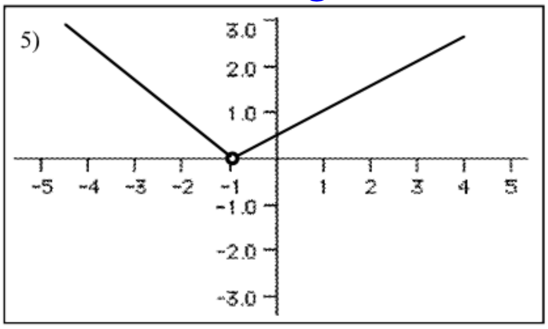
a) $\lim_{x\to2^-} f(x)$ 3 b) $\lim_{x\to2^+} f(x)$ 0 c) $\lim_{x\to2} f(x)$ Dne
d) $f(2)$ 3 e) $\lim_{x\to-\infty} f(x)$ 0 f) $\lim_{x\to\infty} f(x)$ $-\infty$

3) 

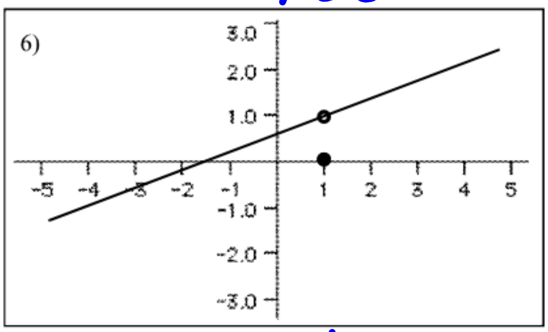
a) $\lim_{x\to3^-} f(x)$ 1 b) $\lim_{x\to3^+} f(x)$ 1 c) $\lim_{x\to3} f(x)$ 1
d) $f(3)$ 1 e) $\lim_{x\to-\infty} f(x)$ $-\infty$ f) $\lim_{x\to\infty} f(x)$ $+\infty$

4) 

a) $\lim_{x\to0^-} f(x)$ 13 b) $\lim_{x\to0^+} f(x)$ 13 c) $\lim_{x\to0} f(x)$ 13
d) $f(0)$ 13 e) $\lim_{x\to-\infty} f(x)$ $+\infty$ f) $\lim_{x\to\infty} f(x)$ $+\infty$

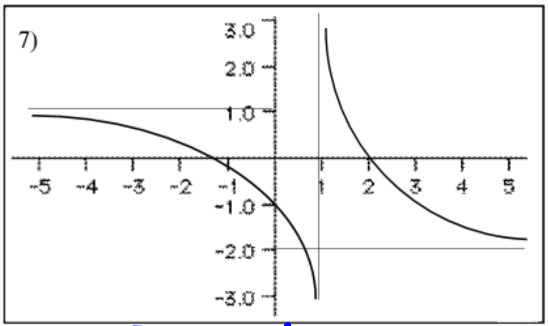
5) 

a) $\lim_{x\to-1^-} f(x)$ 0 b) $\lim_{x\to-1^+} f(x)$ 0 c) $\lim_{x\to-1} f(x)$ 0
d) $f(-1)$ und e) $\lim_{x\to-\infty} f(x)$ $+\infty$ f) $\lim_{x\to\infty} f(x)$ $+\infty$

6) 

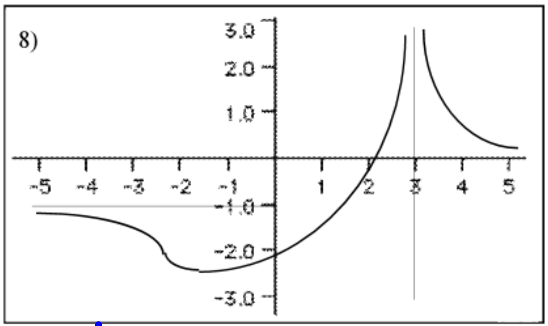
a) $\lim_{x\to1^-} f(x)$ 1 b) $\lim_{x\to1^+} f(x)$ 1 c) $\lim_{x\to1} f(x)$ 1
d) $f(1)$ 0 e) $\lim_{x\to-\infty} f(x)$ $-\infty$ f) $\lim_{x\to\infty} f(x)$ $+\infty$

7)



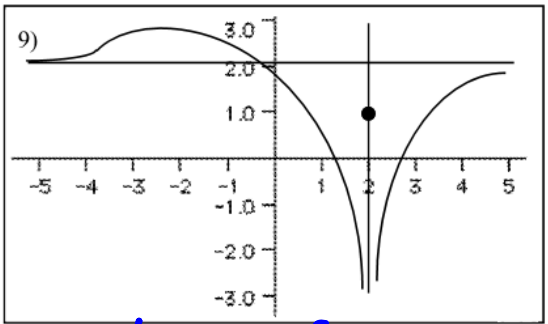
a) $\lim_{x\to1^-} f(x)$ $-\infty$ b) $\lim_{x\to1^+} f(x)$ $+\infty$ c) $\lim_{x\to1} f(x)$ dne
d) $f(1)$ und e) $\lim_{x\to-\infty} f(x)$ 1 f) $\lim_{x\to\infty} f(x)$ -2

8)



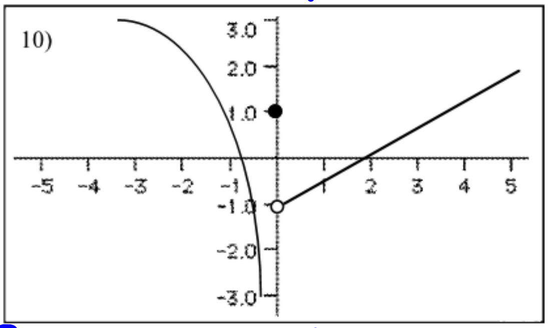
a) $\lim_{x\to3^-} f(x)$ $+\infty$ b) $\lim_{x\to3^+} f(x)$ $+\infty$ c) $\lim_{x\to3} f(x)$ $+\infty$
d) $f(3)$ und e) $\lim_{x\to-\infty} f(x)$ -1 f) $\lim_{x\to\infty} f(x)$ 0

9)



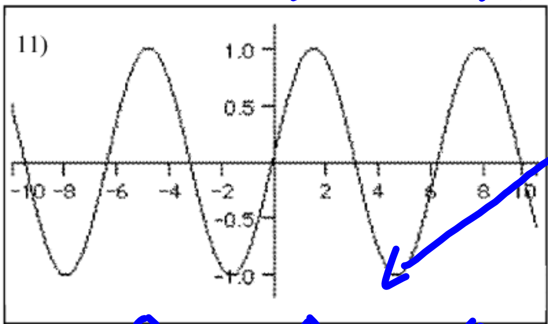
a) $\lim_{x\to2^-} f(x)$ $-\infty$ b) $\lim_{x\to2^+} f(x)$ $-\infty$ c) $\lim_{x\to2} f(x)$ $-\infty$
d) $f(2)$ 1 e) $\lim_{x\to-\infty} f(x)$ 2 f) $\lim_{x\to\infty} f(x)$ 2

10)



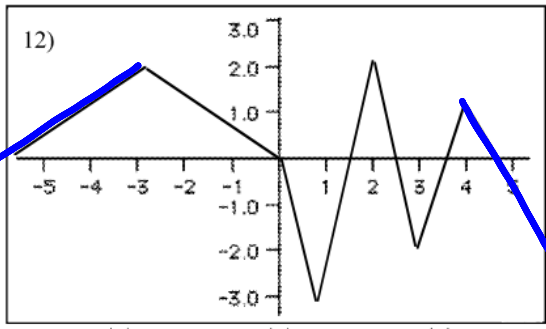
a) $\lim_{x\to0^-} f(x)$ $-\infty$ b) $\lim_{x\to0^+} f(x)$ -1 c) $\lim_{x\to0} f(x)$ dne
d) $f(0)$ 1 e) $\lim_{x\to-\infty} f(x)$ $+\infty$ f) $\lim_{x\to\infty} f(x)$ $+\infty$

11)



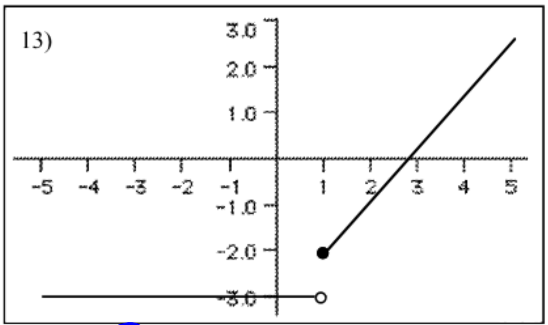
a) $\lim_{x\to0^-} f(x)$ 0 b) $\lim_{x\to0^+} f(x)$ 0 c) $\lim_{x\to0} f(x)$ 0
d) $f(0)$ 0 e) $\lim_{x\to-\infty} f(x)$ dne f) $\lim_{x\to\infty} f(x)$ dne

12)



a) $\lim_{x\to0^-} f(x)$ 0 b) $\lim_{x\to0^+} f(x)$ 0 c) $\lim_{x\to0} f(x)$ 0
d) $f(0)$ 0 e) $\lim_{x\to-\infty} f(x)$ $-\infty$ f) $\lim_{x\to\infty} f(x)$ $-\infty$

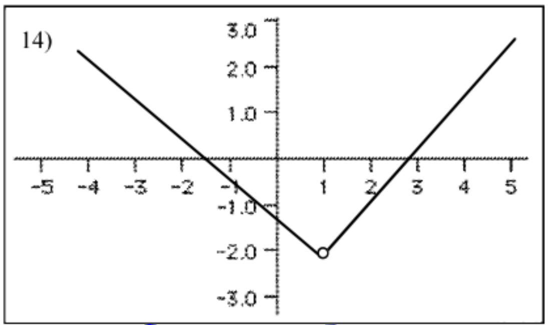
13)



a) $\lim_{x\to 1^-} f(x)$ -3 b) $\lim_{x\to 1^+} f(x)$ -2 c) $\lim_{x\to 1} f(x)$ dne

d) $f(1)$ -2 e) $\lim_{x\to -\infty} f(x)$ -3 f) $\lim_{x\to \infty} f(x)$ $+\infty$

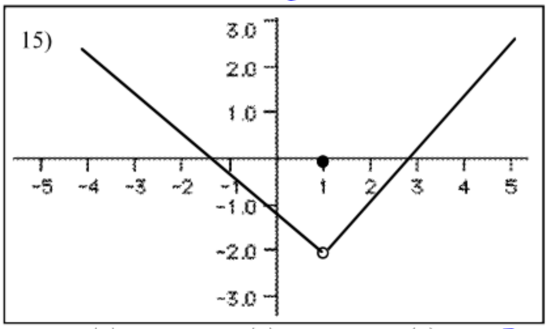
14)



a) $\lim_{x\to 1^-} f(x)$ -2 b) $\lim_{x\to 1^+} f(x)$ -2 c) $\lim_{x\to 1} f(x)$ -2

d) $f(1)$ und e) $\lim_{x\to -\infty} f(x)$ $+\infty$ f) $\lim_{x\to \infty} f(x)$ $+\infty$

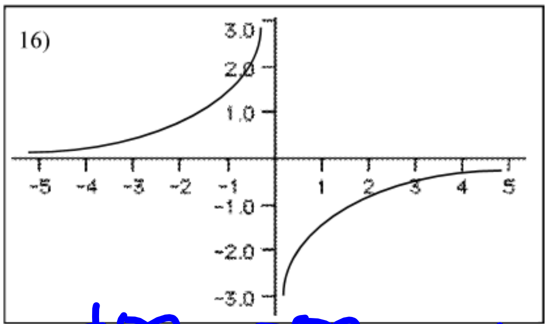
15)



a) $\lim_{x\to 1^-} f(x)$ -2 b) $\lim_{x\to 1^+} f(x)$ -2 c) $\lim_{x\to 1} f(x)$ -2

d) $f(1)$ 0 e) $\lim_{x\to -\infty} f(x)$ $+\infty$ f) $\lim_{x\to \infty} f(x)$ $+\infty$

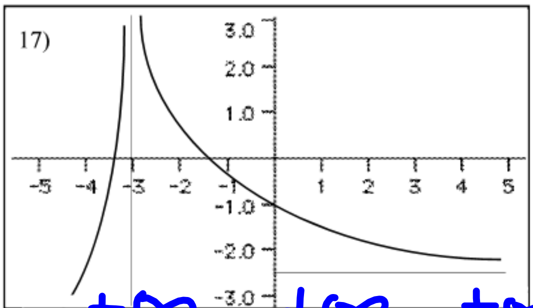
16)



a) $\lim_{x\to 0^-} f(x)$ $+\infty$ b) $\lim_{x\to 0^+} f(x)$ $-\infty$ c) $\lim_{x\to 0} f(x)$ dne

d) $f(0)$ und e) $\lim_{x\to -\infty} f(x)$ 0 f) $\lim_{x\to \infty} f(x)$ 0

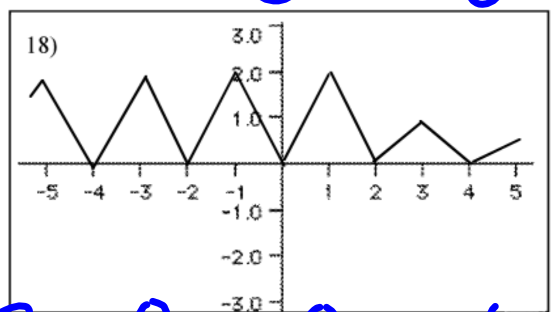
17)



a) $\lim_{x\to -3^-} f(x)$ $+\infty$ b) $\lim_{x\to -3^+} f(x)$ $+\infty$ c) $\lim_{x\to -3} f(x)$ $+\infty$

d) $f(-3)$ und e) $\lim_{x\to -\infty} f(x)$ $-\infty$ f) $\lim_{x\to \infty} f(x)$ -2.5

18)



a) $\lim_{x\to 0^-} f(x)$ 0 b) $\lim_{x\to 0^+} f(x)$ 0 c) $\lim_{x\to 0} f(x)$ 0

d) $f(0)$ 0 e) $\lim_{x\to -\infty} f(x)$ $-\infty$ f) $\lim_{x\to \infty} f(x)$ $+\infty$