Finding Limits Algebraically - Classwork

We are going to now determine limits without benefit of looking at a graph, that is $\lim_{x \to a} f(x)$.

There are three steps to remember:
1) plug in a
2) Factor/cancel and go back to step 1
3) ∞, -∞, or DNE

Example 1) find $\lim_{x \to -2} x^2 - 4x + 1$

You can do this by plugging in.

$\boxed{13}$

Example 2) find $\lim_{x \to -2} \frac{2x-6}{x-2}$

You can also do this by plugging in.

$\boxed{\frac{-10}{-4} = \frac{5}{2}}$

Example 3) find $\lim_{x \to -2} \frac{x^2 - 2x - 8}{x^2 - 4}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

$$\lim_{x \to -2} \frac{x^2 - 2x - 8}{x^2 - 4} = \lim_{x \to -2} \frac{(x-4)(x+2)}{(x-2)(x+2)}$$
$$= \lim_{x \to -2} \frac{(x-4)}{(x-2)} = \frac{-3}{-2} = \frac{3}{2}$$

Example 4) find $\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^3 - 1}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

$$\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^3 - 1} = \lim_{x \to 1} \frac{(x-1)(x-1)}{(x-1)(x^2 + x + 1)}$$
$$= \lim_{x \to 1} \frac{(x-1)}{(x^2 + x + 1)} = 0$$

If steps 1 and 2 do not work (you have a zero in the denominator, your answer is one of the following:

∞	-∞	**Does Not Exist (DNE)**

To determine which, you must split your limit into two separate limits.: $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a+} f(x)$. Make a sign chart by plugging in a number close to a on the left side and determining its sign. You will also plug in a number close to a on the right side and determine its sign. **Each of these will be some form of ∞, either positive or negative.** Only if they are the same will the limit be ∞ or -∞.

What this says is that in this case, $\lim_{x \to a^-} f(x) =$ some form of ∞ and $\lim_{x \to a+} f(x) =$ some form of ∞
You need to check whether they are the same.

Example 5) find $\lim_{x \to 2} \frac{2x+5}{x-2}$

Step 1) Plug in - $\frac{9}{0}$ - no good Step 2) - No factoring/cancel So your answer is ∞, -∞ or DNE

$$\lim_{x \to 2^-} \frac{2x+5}{x-2} = \frac{-}{-}\infty \quad \lim_{x \to 2^+} \frac{2x+5}{x-2} = \frac{-}{+}\infty \quad \therefore \lim_{x \to 2} \frac{2x+5}{x-2} \text{ DNE}$$

Example 6) find $\lim_{x \to 0} \frac{4}{x^2}$

Step 1) Plug in - $\frac{4}{0}$ - no good Step 2) - No factoring/cancel So your answer is ∞, $-\infty$ or DNE

$$\lim_{x \to 0^-} \frac{4}{x^2} = \frac{+}{+}\infty \quad \lim_{x \to 0^-} \frac{4}{x^2} = \frac{+}{+}\infty \quad \therefore \lim_{x \to 0} \frac{4}{x^2} = \infty$$

Example 7) find $\lim_{x \to -3} \frac{x^2+2x-3}{x^2+6x+9}$

$$\lim_{x \to -3} \frac{x^2+2x-3}{x^2+6x+9} = \lim_{x \to -3^-} \frac{x-1}{x+3} = \frac{-}{-}\infty$$

$$\lim_{x \to -3^+} \frac{x-1}{x+3} = \frac{-}{+}\infty \quad \therefore \lim_{x \to -3} \frac{x-1}{x+3} DNE$$

Example 8) find $\lim_{x \to 2} \frac{2x-4}{x^3-6x^2+12x-8}$

$$\lim_{x \to 2} \frac{2x-4}{x^3-6x^2+12x-8} = \lim_{x \to 2} \frac{2(x-2)}{(x-2)^3} =$$

$$\lim_{x \to 2^-} \frac{2}{(x-2)^2} = \frac{+}{+}\infty \quad \lim_{x \to 2^+} \frac{2}{(x-2)^2} = \frac{+}{+}\infty$$

$$\therefore \lim_{x \to 2} \frac{2x-4}{x^3-6x^2+12x-8} = \infty$$

Example 9) $f(x) = \begin{cases} x^2-4, x \ge 1 \\ -2x-1, x < 1 \end{cases}$ find $\lim_{x \to 1} f(x)$

$$\lim_{x \to 1^-} f(x) = -3 \quad \lim_{x \to 1^+} f(x) = -3$$

$$\therefore \lim_{x \to 1} f(x) = -3$$

Example 10) $f(x) = \begin{cases} \frac{x}{x-2}, x \ge 2 \\ \frac{x-3}{x-2}, x < 2 \end{cases}$ find $\lim_{x \to 2} f(x)$

$$\lim_{x \to 2^-} f(x) = \frac{-}{-}! \quad \lim_{x \to 2^+} f(x) = \frac{+}{+}!$$

$$\therefore \lim_{x \to 2} f(x) = \infty$$

Example 11) $$\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}\left(\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right) = \lim_{x \to 0} \frac{x}{x\left(\sqrt{x+2}+\sqrt{2}\right)} = \lim_{x \to 0} \frac{1}{\left(\sqrt{x+2}+\sqrt{2}\right)} = \frac{1}{2\sqrt{2}}$$

Finally, we are interested also in problems of the type: $\lim_{x \to \pm\infty} f(x)$. Here are the rules:

Write $f(x)$ as a fraction.

1) If the highest power of x appears in the denominator (bottom heavy), $\lim_{x \to \pm\infty} f(x) = 0$
2) If the highest power of x appears in the numerator (top heavy), $\lim_{x \to \pm\infty} f(x) = \pm\infty$ plug in very large or small numbers and determine the sign of the answer
3) If the highest power of x appears both in the numerator and denominator (powers equal), $\lim_{x \to \pm\infty} f(x) = \frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$

Example 12) $\lim_{x \to \infty} \frac{4x^2+50}{x^3-85}$

$\boxed{0}$

Example 13) $\lim_{x \to \infty} \frac{4x^3-5x^2+3x-1}{5x^3-7x-25}$

$\boxed{\frac{4}{5}}$

Example 14) $\lim_{x \to \infty} \frac{3x^3-23}{4x-1}$

$\boxed{\infty}$

Example 15) $\lim_{x \to -\infty} \frac{4x-5x^2+3}{\frac{1}{x}}$

$\boxed{\infty}$

Example 16) $\lim_{x \to \infty} \frac{\sqrt{x^2-3x}}{2x+1}$

$\boxed{\frac{1}{2}}$

Example 17) $\lim_{x \to -\infty} \frac{\sqrt{x^2-3x}}{2x+1}$

$\boxed{-\frac{1}{2}}$

Finding Limits Algebraically - Homework

1) $\lim\limits_{x\to 5} 12$

$\boxed{12}$

2) $\lim\limits_{x\to 0} \pi$

$\boxed{\pi}$

3) $\lim\limits_{x\to 2} 4x$

$\boxed{8}$

4) $\lim\limits_{x\to 5} 3x^2 - 4x - 1$

$\boxed{54}$

5) $\lim\limits_{x\to 0^-} 5x^3 - 7x^2 + 2^x - 2$

$\boxed{-1}$

6) $\lim\limits_{y\to -1} 3y^4 - 6y^3 - 2y$

$\boxed{11}$

7) $\lim\limits_{x\to 4} \dfrac{2x-4}{x-1}$

$\boxed{\dfrac{4}{3}}$

8) $\lim\limits_{x\to -2} \dfrac{x^2+4x+4}{x^2}$

$\boxed{0}$

9) $\lim\limits_{x\to 1} \dfrac{2x-2}{x-1}$

$\boxed{\lim\limits_{x\to 1} \dfrac{2(x-1)}{x-1} = 2}$

7) $\lim\limits_{x\to 4} \dfrac{2x-4}{x-1}$

$\boxed{\lim\limits_{x\to 4} \dfrac{(x+4)(x-4)}{x-4} = 8}$

8) $\lim\limits_{x\to -2} \dfrac{x^2+4x+4}{x^2}$

$\boxed{\lim\limits_{t\to -2} \dfrac{(t+2)(t^2-2t+4)}{t+2} = 12}$

9) $\lim\limits_{x\to 1} \dfrac{2x-2}{x-1}$

$\boxed{\lim\limits_{x\to 2} \dfrac{(x-2)^2}{(x-3)(x+2)} = 0}$

10) $\lim\limits_{x\to 4} \dfrac{x^2-16}{x-4}$

$\boxed{\lim\limits_{x\to 4} \dfrac{(x+4)(x-4)}{x-4} = 8}$

11) $\lim\limits_{t\to -2} \dfrac{t^3+8}{t+2}$

$\boxed{\lim\limits_{t\to -2} \dfrac{(t+2)(t^2-2t+4)}{t+2} = 12}$

12) $\lim\limits_{x\to 2} \dfrac{x^2-4x+4}{x^2+x-6}$

$\boxed{\lim\limits_{x\to 2} \dfrac{(x-2)(x-2)}{(x+3)(x-2)} = 0}$

13) $\lim\limits_{x\to -1} \dfrac{x^2+6x+5}{x^2-3x-4}$

$\boxed{\lim\limits_{x\to -1} \dfrac{(x+5)(x+1)}{(x-4)(x+1)} = \dfrac{-4}{5}}$

14) $\lim\limits_{x\to 1} \dfrac{x^3+x^2-5x+3}{x^3-3x+2}$

$\boxed{\lim\limits_{x\to 1} \dfrac{(x-1)^2(x+3)}{(x-1)^2(x+2)} = \dfrac{4}{3}}$

15) $\lim\limits_{x\to 3} \dfrac{x}{x-3}$

$\lim\limits_{x\to 3^+} \dfrac{x}{x-3} = \infty \quad \lim\limits_{x\to 3^-} \dfrac{x}{x-3} = -\infty$

$\lim\limits_{x\to 3^+} \dfrac{x}{x-3} = DNE$

16) $\lim\limits_{x\to 5} \dfrac{x}{x^2-25}$

$\lim\limits_{x\to 5^+} \dfrac{x}{x^2-25} = \infty \quad \lim\limits_{x\to 5^-} \dfrac{x}{x^2-25} = -\infty$

$\lim\limits_{x\to 5} \dfrac{x}{x^2-25} = DNE$

17) $\lim\limits_{y\to 6} \dfrac{y+6}{y^2-36}$

$\lim\limits_{x\to 6^+} \dfrac{1}{x-6} = \infty \quad \lim\limits_{x\to 6^-} \dfrac{1}{x-6} = -\infty$

$\lim\limits_{x\to 6} \dfrac{1}{x-6} = DNE$

18) $\lim\limits_{x\to 4} \dfrac{3-x}{x^2-2x-8}$

$\lim\limits_{x\to 4^+} \dfrac{3-x}{(x-4)(x+2)} = -\infty$

$\lim\limits_{x\to 4^-} \dfrac{3-x}{(x-4)(x+2)} = \infty$

$\lim\limits_{x\to 4} \dfrac{3-x}{(x-4)(x+2)} = DNE$

19) $\lim\limits_{x\to 1} \dfrac{4}{x^2-2x+1}$

$\lim\limits_{x\to 1^+} \dfrac{4}{(x-1)^2} = \infty \quad \lim\limits_{x\to 1^-} \dfrac{4}{(x-1)^2} = \infty$

$\lim\limits_{x\to 1} \dfrac{4}{(x-1)^2} = \infty$

20) $\lim\limits_{x\to 5} \dfrac{x}{|x-5|}$

$\lim\limits_{x\to 5^+} \dfrac{x}{|x-5|} = \infty \quad \lim\limits_{x\to 5^-} \dfrac{x}{|x-5|} = \infty$

$\lim\limits_{x\to 5} \dfrac{x}{|x-5|} = \infty$

21) $\lim\limits_{x\to 3} \dfrac{-x^2}{x^2-6x+9}$

$\lim\limits_{x\to 3^+} \dfrac{-x^2}{(x-3)^2} = -\infty \quad \lim\limits_{x\to 3^-} \dfrac{-x^2}{(x-3)^2} = -\infty$

$\lim\limits_{x\to 3} \dfrac{-x^2}{(x-3)^2} = -\infty$

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Finding Limits Algebraically - Homework

1) $\lim_{x \to 5} 12$ = 12

2) $\lim_{x \to 0} \pi$ = π

3) $\lim_{x \to 2} 4x$ = 8

4) $\lim_{x \to 5} 3x^2 - 4x - 1$

$3(5)^2 - 4(5) - 1 = 54$

5) $\lim_{x \to 0^-} 5x^3 - 7x^2 + 2^{x} - 2$

$= -1$?

6) $\lim_{y \to -1} 3y^4 - 6y^3 - 2y$

$3(-1)^4 - 6(-1)^3 - 2(-1) = 11$

7) $\lim_{x \to 4} \frac{2x-4}{x-1}$ $= \frac{2(4)-4}{4-1} = \frac{4}{3}$

8) $\lim_{x \to -2} \frac{x^2+4x+4}{x^2}$ $= \frac{0}{4} = 0$

$\frac{(-2)^2 + (4)(-2) + 4}{(-2)^2}$

9) $\lim_{x \to 1} \frac{2x-2}{x-1}$ $= \frac{2(x-1)}{x-1} = 2$

10) $\lim_{x \to 4} \frac{x^2-16}{x-4}$ $= \frac{(x-4)(x+4)}{x-4}$, $4+4 = 8$

11) $\lim_{t \to -2} \frac{t^3+8}{t+2}$ $= \frac{(t+2)(t^2-2t+4)}{t+2} = 12$

12) $\lim_{x \to 2} \frac{x^2-4x+4}{x^2+x-6}$ $= \frac{(x-2)(x-2)}{(x-2)(x+3)} = \frac{(2-2)}{(2+3)} = \frac{0}{5} = 0$

13) $\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4}$ $= \frac{(x+5)(x+1)}{(x-4)(x+1)} = \frac{-1+5}{-1-4} = \frac{4}{-5}$

14) $\lim_{x \to 1} \frac{x^3+x^2-5x+3}{x^3-3x+2}$ $= \frac{4}{3}$

ck calculator table

15) $\lim_{x \to 3} \frac{x}{x-3}$ $= \frac{3}{0} =$ DNE

x	2.9999	3	3.0001
y	-29999		30001

16) $\lim_{x \to 5} \frac{x}{x^2-25}$ dne

ck table in calculator

17) $\lim_{y \to 6} \frac{y+6}{y^2-36}$ $= \frac{y+6}{(y+6)(y-6)} = \frac{1}{0}$ = DNE

x	5.9999	6	6.0001
y	-10000		10000

18) $\lim_{x \to 4} \frac{3-x}{x^2-2x-8}$ $= \frac{3-x}{(x-4)(x+2)}$

ck table in calculator dne

19) $\lim_{x \to 1} \frac{4}{x^2-2x+1}$ $= \frac{4}{0} = +\infty$

$1 - 2 + 1$

x	.9999	1	1.0001
y	4x10^8		4x10^8

20) $\lim_{x \to 5} \frac{x}{|x-5|}$ $= +\infty$

ck table in calculator

21) $\lim_{x \to 3} \frac{-x^2}{x^2-6x+9}$ $= -\infty$

x	2.9999	3	3.0001
y	-9x10^8		-9x10^8

$(-1)^3-1 = -2$

22) $f(x)=\begin{cases} x-1, x\ge 3 \\ 2x-3, x<3 \end{cases}$ find $\lim\limits_{x\to 3} f(x)$ dne

$3-1=2$ not the same per graph

$2(3)-3=3$

$6-3$

23) $f(x)=\begin{cases} x^3-1, x\ge -1 \\ 2x, x<-1 \end{cases}$ find $\lim\limits_{x\to -1} f(x)$ $=-2$

$2(-1)=-2$

24) $f(x)=\begin{cases} \dfrac{x-2}{x-1}, x\ge 1 \\ \dfrac{x}{x-1}, x<1 \end{cases}$ find $\lim\limits_{x\to 1} f(x)$ $-\infty$

$\frac{1-2}{1-1}=\frac{-}{0}$ und

$\frac{1}{1-1}=\frac{}{0}$ und

per graph

25) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$ $\dfrac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)} = \dfrac{x+4-4}{x(\sqrt{x+4}+2)} = x$

$\dfrac{x}{x(\sqrt{x+4}+2)} = \dfrac{1}{\sqrt{x+4}+2} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$

26) Let $f(x)=\begin{cases} x^2-2x-3, x\ne 2 \\ k-3, x=2 \end{cases}$ -3

$4-4-3$

$(2)^2-2(2)-3$

find k such that $\lim\limits_{x\to 2} f(x)=f(2)$ $\begin{cases} x^2-2x-3, x\ne 2 \\ -3, x=2 \end{cases}$

$k=0$

27) $f(x)=\begin{cases} \dfrac{x^2-49}{x-7}, x\ne 7 \\ k^2-2, x=7 \end{cases}$ $\frac{(x+7)(x-7)}{x-7}$

find k such that $\lim\limits_{x\to 7} f(x)=f(7)$ $k=4$

$k^2-2=14 \quad k^2=16$

28) $\lim\limits_{x\to\infty} 6$

29) $\lim\limits_{x\to\infty}(-2x+11)$

30) $\lim\limits_{x\to -\infty}(3x^4-3x^3+5x^2+8x-3)$

31) $\lim\limits_{x\to\infty}\dfrac{2x-3}{4x+5}$

32) $\lim\limits_{x\to -\infty}\dfrac{7-3x^3}{2x^3+1}$

33) $\lim\limits_{x\to\infty}\dfrac{2}{5x-3}$

34) $\lim\limits_{x\to -\infty}\dfrac{2x+30}{6x^{12}-5}$

35) $\lim\limits_{x\to\infty}\dfrac{4x^4}{6x^3-19}$

36) $\lim\limits_{x\to -\infty}\dfrac{4x^2-3x-2-5x^3}{9x^2+9x+7}$

37) $\lim\limits_{x\to\infty}\dfrac{x}{\sqrt{x^2+4}}$

38) $\lim\limits_{x\to -\infty}\dfrac{x}{\sqrt{x^2+4}}$

39) $\lim\limits_{x\to -\infty}\dfrac{\sqrt{3x^2+x}}{x^2-1}$