

One sided limits

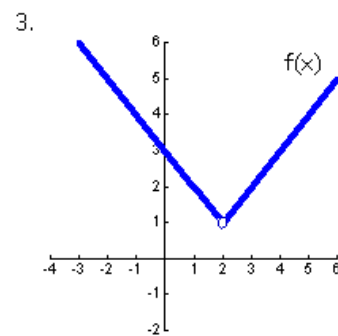
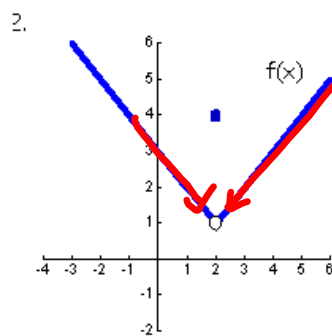
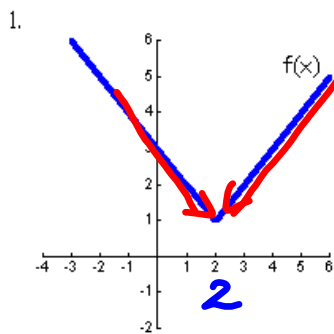
$$\lim_{x \rightarrow a^-} f(x) = L$$

"The limit of $f(x)$ as x approaches a from the left."

$$\lim_{x \rightarrow a^+} f(x) = L$$

"The limit of $f(x)$ as x approaches a from the right."

Find $\lim_{x \rightarrow 2} (f(x))$



$$\lim_{x \rightarrow 2^+} f(x) = \quad |$$

$$\lim_{x \rightarrow 2^-} f(x) = \quad |$$

$$\lim_{x \rightarrow 2} f(x) = \quad |$$

$$\lim_{x \rightarrow 2^+} f(x) = \quad |$$

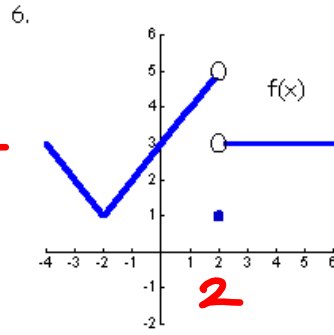
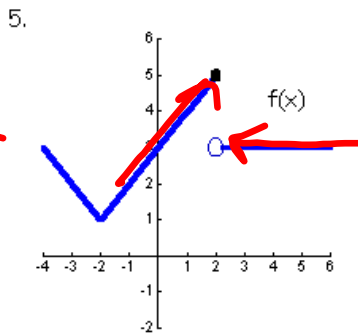
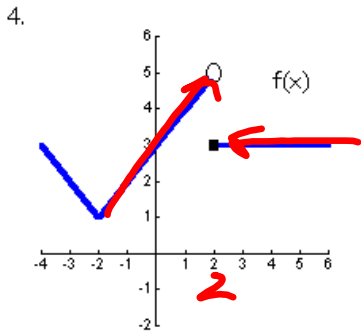
$$\lim_{x \rightarrow 2^-} f(x) = \quad |$$

$$\lim_{x \rightarrow 2} f(x) = \quad |$$

$$\lim_{x \rightarrow 2^+} f(x) = \quad |$$

$$\lim_{x \rightarrow 2^-} f(x) = \quad |$$

$$\lim_{x \rightarrow 2} f(x) = \quad |$$



$$\lim_{x \rightarrow 2^+} f(x) = 3$$

$$\lim_{x \rightarrow 2^-} f(x) = 5$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 2^+} f(x) = 3$$

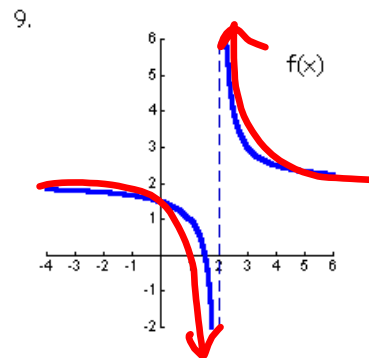
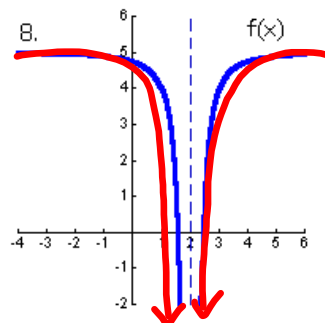
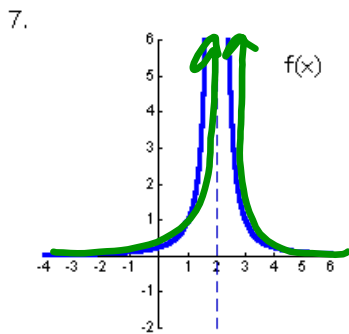
$$\lim_{x \rightarrow 2^-} f(x) = 5$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 2^+} f(x) = 3$$

$$\lim_{x \rightarrow 2^-} f(x) = 5$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$



$$\lim_{x \rightarrow 2^+} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = +\infty$$

$$\lim_{x \rightarrow 2} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

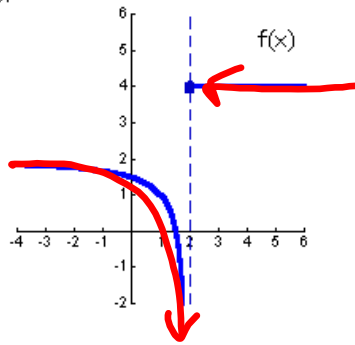
$$\lim_{x \rightarrow 2} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

10.

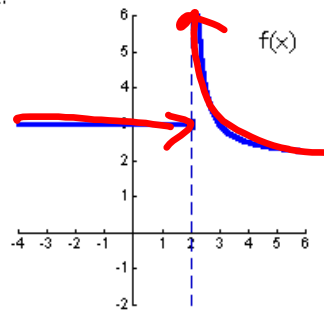


$$\lim_{x \rightarrow 2^+} f(x) = 4$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

11.

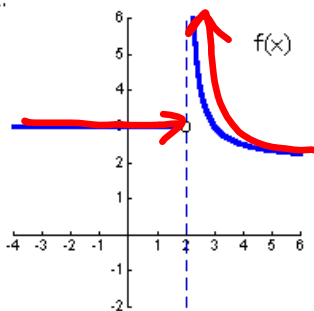


$$\lim_{x \rightarrow 2^+} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = 3$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

12.

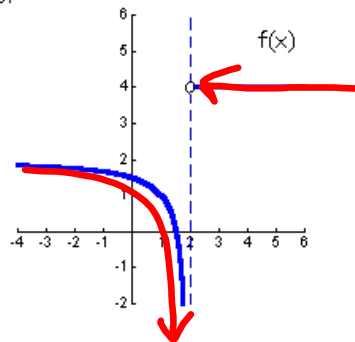


$$\lim_{x \rightarrow 2^+} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = 3$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

13.



$$\lim_{x \rightarrow 2^+} f(x) = 4$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

Example

Evaluate the limits graphically then numerically.

a) Graphically.

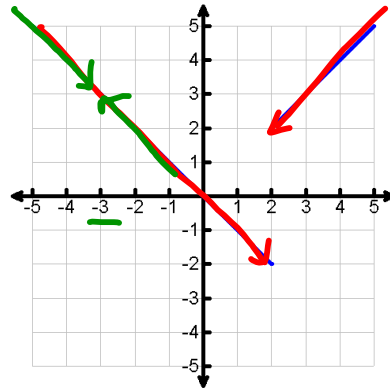
$$g(x) = \frac{x|x-2|}{x-2}$$

$$\lim_{x \rightarrow 2^+} g(x) = 2$$

$$\lim_{x \rightarrow 2^-} g(x) = -2$$

$$\lim_{x \rightarrow -3^-} g(x) = 3$$

$$\lim_{x \rightarrow -3^+} g(x) = 3$$



b) Numerically.

$$g(x) = \frac{x|x-2|}{x-2}$$

$$\lim_{x \rightarrow 2^+} g(x) = 2$$

$$\lim_{x \rightarrow 2^-} g(x) = -2$$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$g(x)$	-1.9	-1.99	-1.999	und	2.001	2.01	2.1

$\xrightarrow{\hspace{10em}}$

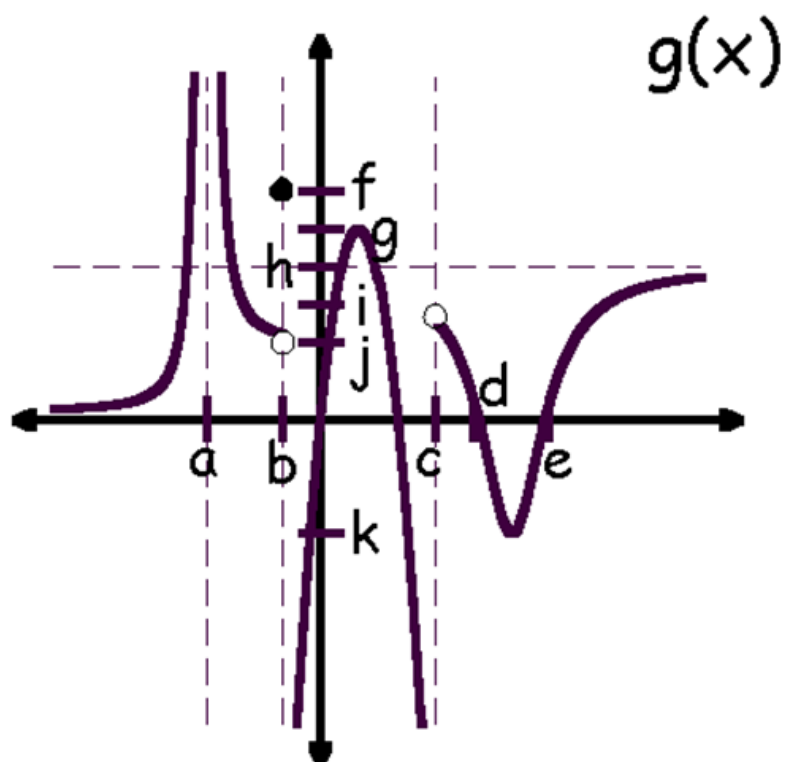
$$\lim_{x \rightarrow -3^-} g(x) = 3$$

$$\lim_{x \rightarrow -3^+} g(x) = 3$$

x	-3.001	-3	-2.999
$g(x)$	3.001	3	2.999

$\xrightarrow{\hspace{10em}}$

Piecewise and one-sided limits:
p.76: #20



p. 76 #5-10 all, 11, 13, 15, 19, 21
Limits Worksheet