

Warm-Up

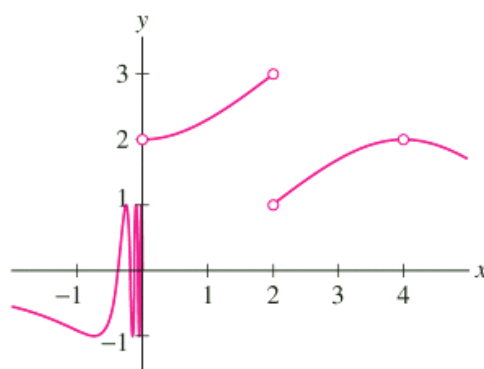


FIGURE 7

Evaluate the following. If a limit DNE, explain why.

1. $\lim_{x \rightarrow 2} f(x) = \text{dne}$

4. $f(4) = \text{und}$

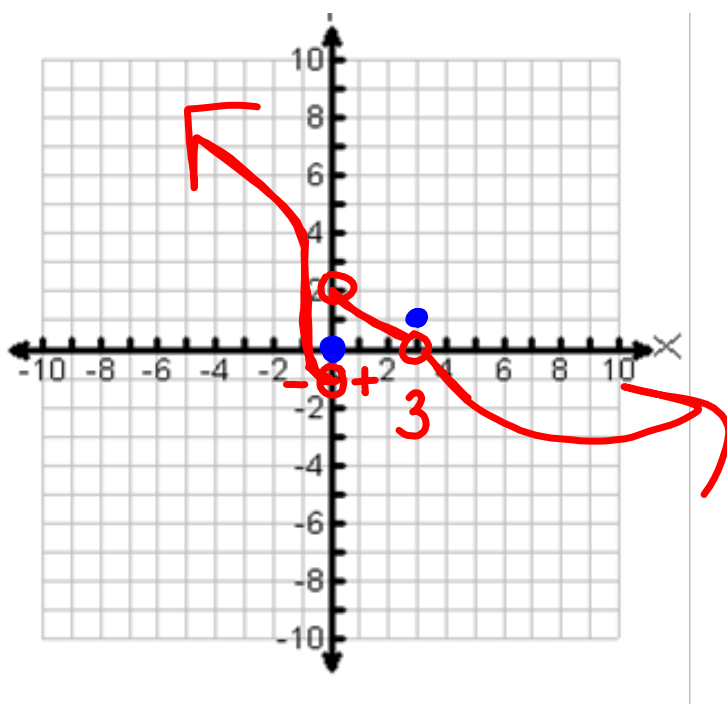
2. $f(2) = \text{und.}$

5. $\lim_{x \rightarrow 0} f(x) = \text{dne}$

3. $\lim_{x \rightarrow 4} f(x)$

Warm-up: 1.12.22

- 1) Pick up two handouts on back table and get supplies. Glue pages into INB.
- 2) Get out HW and calendar. Get calculator.
- 3) Complete sketch at the bottom of pg. 23 in the packet for warm-up.



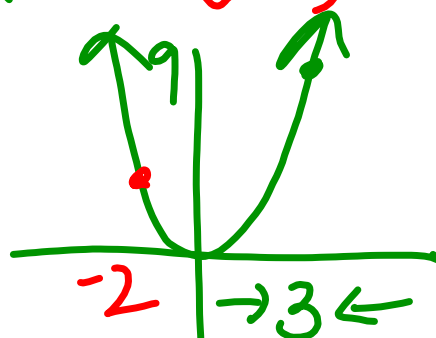
Properties of Limits

1. $\lim_{x \rightarrow c} a = a$

$$\lim_{x \rightarrow -2} x^2 = (-2)^2 = 4$$

2. $\lim_{x \rightarrow c} x = c$

$$x \rightarrow -2$$

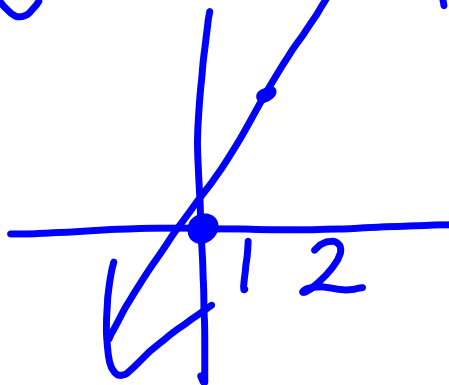


3. $\lim_{x \rightarrow c} x^n = c^n$

4. $\lim_{x \rightarrow c} ax = ac$

$$\lim_{x \rightarrow 2} 5x = 5(2) = \underline{10}$$

$$y = \frac{5}{1}x$$



THEOREM 1 Basic Limit Laws Assume that $\lim_{x \rightarrow c} f(x)$ and $\lim_{x \rightarrow c} g(x)$ exist. Then:

(i) **Sum Law:**

$$\lim_{x \rightarrow c} (f(x) + g(x)) = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x)$$

(ii) **Constant Multiple Law:** For any number k ,

$$\lim_{x \rightarrow c} kf(x) = k \lim_{x \rightarrow c} f(x)$$

(iii) **Product Law:**

$$\lim_{x \rightarrow c} f(x)g(x) = \left(\lim_{x \rightarrow c} f(x) \right) \left(\lim_{x \rightarrow c} g(x) \right)$$

(iv) **Quotient Law:** If $\lim_{x \rightarrow c} g(x) \neq 0$, then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}$$

<http://archives.math.utk.edu/visual.calculus/1/limits.18/index.html>



Evaluate the following using the limit laws:

$$1. \lim_{x \rightarrow -3} 3x + 4 = \lim_{x \rightarrow -3} 3x + \lim_{x \rightarrow -3} 4$$

x	y
-3	-5

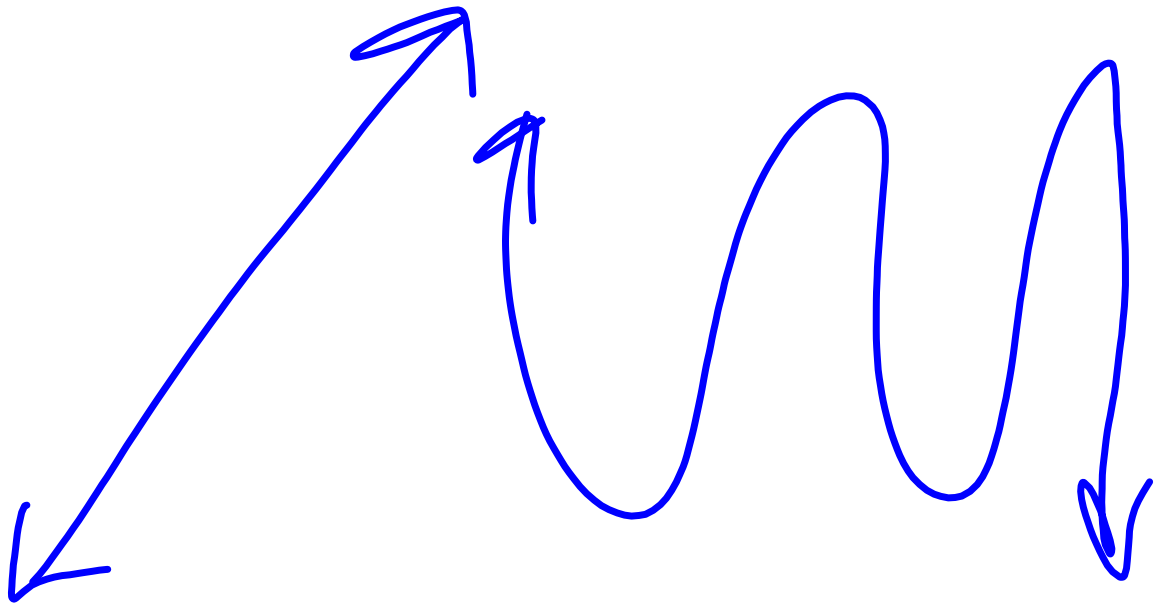
$$3(-3)$$

$$-9 + 4 = -5$$

$$2. \lim_{x \rightarrow 2} x(x+1)(x+2)$$

$$\left(\lim_{x \rightarrow 2} x \right) \left(\lim_{x \rightarrow 2} x+1 \right) \left(\lim_{x \rightarrow 2} x+2 \right)$$

$$2 \cdot 3 \cdot 4 = \underline{24}$$



$$\text{Given } \lim_{x \rightarrow c} f(x) = \underline{5} \text{ and } \lim_{x \rightarrow c} \underline{g(x)} = \underline{-4}$$

Evaluate:

$$1. \lim_{x \rightarrow c} [6g(x)] = 6 \left(\lim_{x \rightarrow c} g(x) \right) = 6 \cdot (-4) = -24$$

$$2. \lim_{x \rightarrow c} [3f(x) - 2g(x)] = 3(5) - 2(-4) = 15 + 8 = 23$$

$$3. \lim_{x \rightarrow c} \left[\frac{f(x)}{g(x)} \right] = \frac{5}{-4} = -\frac{5}{4}$$

Today, we solve limits Algebraically:

*Always try Direct Substitution first

*If it doesn't work then try to simplify algebraically and/or evaluate the limit graphically and numerically.

Finding a limit by substitution

$$\lim_{x \rightarrow 1} x^3 - 4x^2 + 5x - 3 = -1$$

$$(1)^3 - 4(1)^2 + 5(1) - 3 = -1$$

x	y
1	-1

Limit

1) D.S. = direct sub

2) Factor and cancel

When substitution doesn't work, try Algebra

* Your new function is a "function that agrees at all but one point."

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = \frac{(-1)^2 - 1}{-1 + 1} = \frac{(x+1)(x-1)}{x+1} =$$

D.S = und

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = -2$$

$$-1 - 1 = -2$$

Examples:

1. $\lim_{x \rightarrow 3} \frac{x^2 - 1}{x + 1}$

$$2. \lim_{x \rightarrow \frac{3\pi}{2}} \sin x = -1 \quad \left| \quad \lim_{x \rightarrow \frac{2\pi}{3}} \sin x \right.$$

1) D.S. $\sin \frac{3\pi}{2}$
(cos, sin)
x, y

$$= \frac{\sqrt{3}}{2}$$

$$3. \lim_{x \rightarrow -5} \frac{x^2 - 25}{x^3 + 125} = \frac{-2}{15}$$

D.S. = und

$$\frac{\cancel{(x+5)}(x-5)}{\cancel{(x+5)}(x^2-5x+25)}$$
$$\frac{-5-5}{(-5)^2-5(-5)+25} = \frac{-10}{75}$$

$$\textcircled{-} \frac{1-4}{(1)^2 - 6(1) + 8} = \left(\frac{-3}{3} \right) = -1$$
$$1 - 6 + 8 \quad -(-1) = 1$$

pg. 25 omit[#] 10, 15
pg. 26 ALL