$$\lim_{x\to -3} \frac{x^2-4x+3}{(\cancel{x})^2-9} = \text{dne}$$

1) D.S.
2) Factor/cancel

1) D.S. = und

2) $\dfrac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+3)} = \dfrac{-3-1}{-3+3} = \dfrac{-3-1}{0}$

-3.001	-3	-2.999
40,001	und	-39,999

one-sided limits are different

Given

$\lim_{x\to c} f(x) = 2$ and $\lim_{x\to c} g(x) = 3$

Find a) $\lim_{x\to c}[5g(x)]$ (3) $= 15$ b) $\lim_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{2}{3}$

c) $\lim_{x\to c}[g(x)f(x)]$ $= 6$ d) $\lim_{x\to c}[g(x)+f(x)]$ $= 5$

First ~~another special~~ limit:

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1}$$

This will be helpful when trying to algebraically manipulate the limit of certain trig. functions.

-.001	0	.001
1		1

Indeterminate Forms

A function is of indeterminate form if when $f(x)$ is evaluated at $x = c$, you obtain an undefined expression of the type

$$\frac{0}{0},\quad \frac{\infty}{\infty},\quad \infty\cdot 0,\quad \infty-\infty$$

If a function is of indeterminate form, you must transform the limit algebraically and evaluate the limit as x approaches c.

FACTOR THE NUMERATOR AND DENOMINATOR AND CANCEL COMMON FACTORS

Example: $\lim_{x\to 2} \frac{x^2-3x+2}{x-2} = 1$

1) D.S. ⇒ und
2) F.C

$\frac{\cancel{(x-2)}(x-1)}{\cancel{x-2}} = 2-1$

MULTIPLY BY THE CONJUGATE AND CANCEL COMMON FACTORS

Example: $\lim_{x\to 0} \frac{\sqrt{x+16}-4}{x} \cdot \frac{(\sqrt{x+16}+4)}{(\sqrt{x+16}+4)}$

1) D.S. ⇒ und

$\frac{x+16-16}{x(\sqrt{x+16}+4)} = \frac{\cancel{x}}{\cancel{x}(\sqrt{x+16}+4)}$

$= \frac{1}{\sqrt{x+16}+4} = \frac{1}{\sqrt{0+16}+4} = \frac{1}{8}$

Remember our "special" trig. limit...

Example: $\lim_{x\to 0} \frac{\sin x}{5x} = \frac{1}{5}$

Constant multiple

$\frac{1}{5} \cdot \frac{\sin x}{x}$

$\frac{1}{5}\left(\lim_{x\to 0} \frac{\sin x}{x}\right)$

$\frac{1}{5} \cdot 1 = \frac{1}{5}$

GET A COMMON DENOMINATOR AND CANCEL COMMON FACTORS

Example: $\lim_{x\to 3} \frac{1^{(x+3)}}{x-3} - \frac{6}{x^2-9} = \frac{1}{6}$

1) D.S. ⇒ und

$(x+3)(x-3)$

$\frac{x+3}{(x+3)(x-3)} - \frac{6}{(x+3)(x-3)}$

$\frac{x+3-6}{(x+3)(x-3)}$

$\frac{\cancel{x-3}}{(x+3)\cancel{(x-3)}} = \frac{1}{x+3}$

$= \frac{1}{3+3} = \frac{1}{6}$

$$\lim_{x\to 0}\frac{\sin x}{\sqrt[3]{x}}\cdot\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}=\sqrt[3]{x^3}$$

Rationalize the denominator

D.S. ⇒ und

$$=\frac{\sin x\cdot\sqrt[3]{x^2}}{x}$$

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\cdot\left(\lim_{x\to 0}\sqrt[3]{0^2}\right)$$

$$1\cdot 0=0$$

$$\lim_{x\to 0}\frac{\cos x\tan x}{x}$$

D.S. ⇒ und.

$$\frac{\cancel{\cos x}\cdot\frac{\sin x}{\cancel{\cos x}}}{x}$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$\tan x=\frac{\sin x}{\cos x}$

$\sec x=\frac{1}{\cos x}$

$\csc x=\frac{1}{\sin x}$

$\cot x=\frac{\cos x}{\sin x}$

$$\lim_{x\to 0}\frac{\tan^2 x}{x}$$

$$=\lim_{x\to 0}\frac{\tan x}{x}\cdot\tan x=\frac{\frac{\sin x}{\cos x}}{x}\cdot\frac{\sin x}{\cos x}$$

$$\frac{\sin x}{\cos x}\cdot\frac{1}{x}\quad\boxed{\frac{\sin x}{x\cos x}\cdot\frac{\sin x}{\cos x}}$$

$$\frac{\sin x}{x}\cdot\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}$$

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{1}{\cos x}\right)\left(\tan x\right)\text{ or }\frac{\sin x}{\cos x}$$

$$1\cdot\frac{1}{\cos 0}\cdot\frac{\sin 0}{\cos 0}$$

$$1\cdot\frac{1}{1}\cdot\frac{0}{1}$$

$$1\cdot 1\cdot 0=0$$

Reverse Rationalize to Simplify

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

1) D.S. ⇒ und

$$\frac{x+1-4}{\cancel{(x-3)}(\sqrt{x+1}+2)}=\frac{\cancel{x-3}}{1}$$

$$\frac{1}{(\sqrt{3+1}+2}=\frac{1}{4}$$

Sketch a possible graph for a function that satisfies the following conditions:

(a) $\lim_{x\to 0^-} f(x) = 1$

(b) $\lim_{x\to 0^+} f(x) = -1$

(c) $\lim_{x\to 2^-} f(x) = 0$

(d) $\lim_{x\to 2^+} f(x) = 1$

(e) $f(2) = 1$

(f) $f(0)$ is undefined