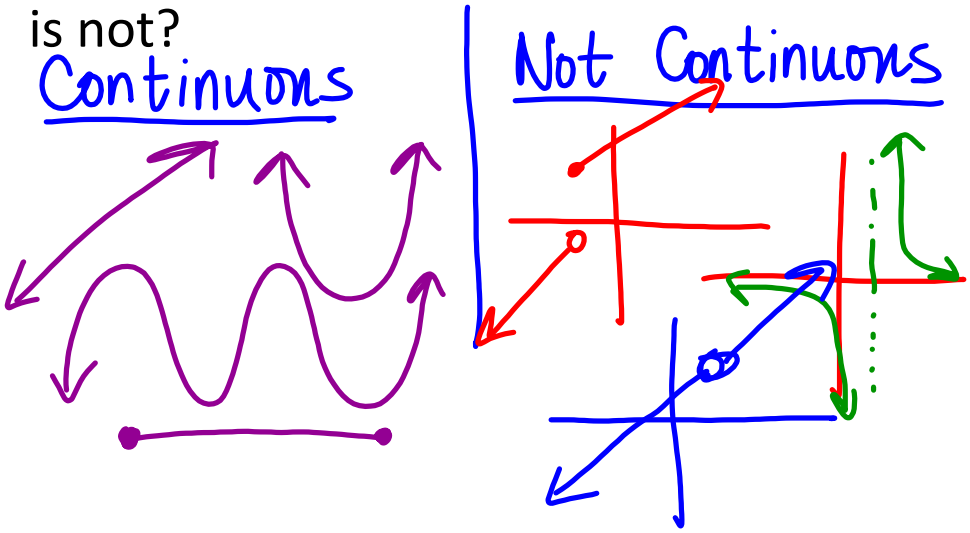
*Brainstorm*: If I said a function was continuous what do you think I mean? Can you sketch a function that you think is continuous? Can you sketch one that is not?




## A little more practice on...

Limits with piecewise functions:

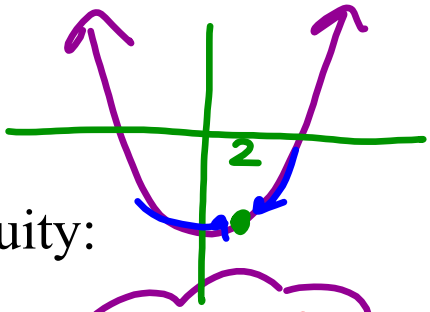
$$f(x)=\begin{cases} x-2, & x\le 0 \\ x^2+1, & x>0 \end{cases}$$

$\lim_{x\to 0^-}$ $0-2=-2$

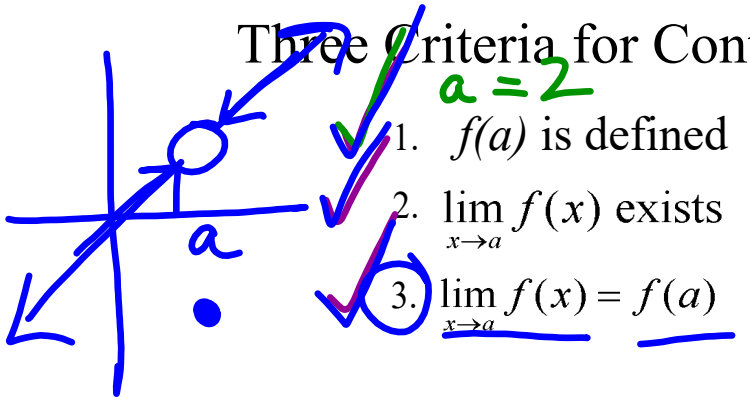
Find $\lim_{x\to 0} f(x)$ = dne

$\lim_{x\to 0^+}$ $0^2+1=1$

# 1.4 Continuity



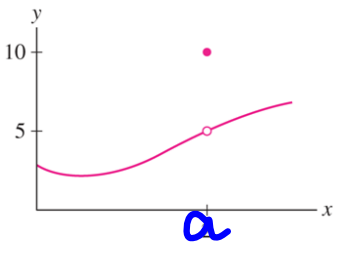

## Three Criteria for Continuity:




1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Remember these 3 things!

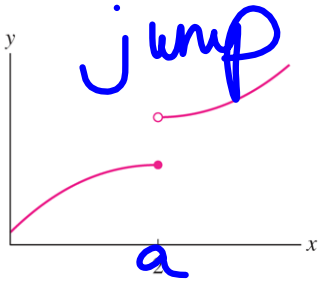
**Example** Determine which criteria are True.




$f(a)$ is defined T

$\lim_{x \to a} f(x)$ exists T

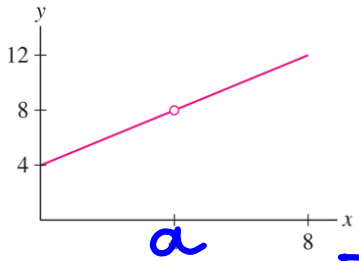
$\lim_{x \to a} f(x) = f(a)$ F




$f(a)$ is defined T

$\lim_{x \to a} f(x)$ exists F

$\lim_{x \to a} f(x) = f(a)$ F



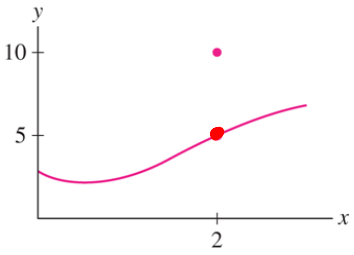

$f(a)$ is defined F

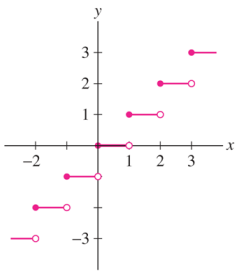
$\lim_{x \to a} f(x)$ exists T

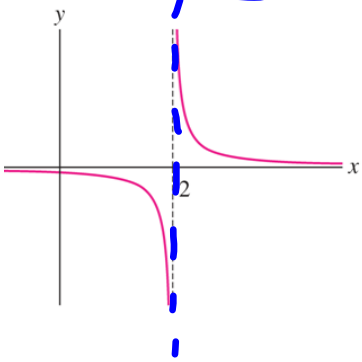
$\lim_{x \to a} f(x) = f(a)$ F

There are three *main* kinds of discontinuities:

1). hole/removable 2). jump 3). infinite

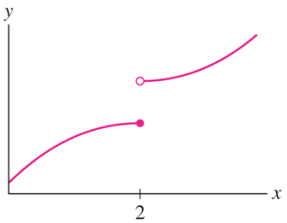


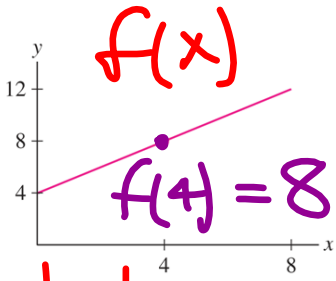




Jump and Infinite discontinuities are non-removable.

Example State the type of discontinuity. If it's removable, find the value that will make the function continuous.



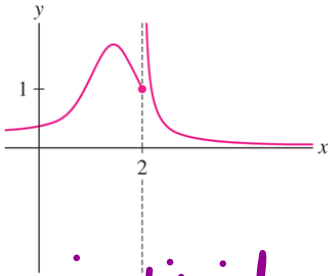

jump




f(x)

f(4) = 8

hole removable




infinite

Is the function continuous? NO If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

$x=2$

$$f(x)=\frac{x^2-4}{x-2}$$

look for den = 0

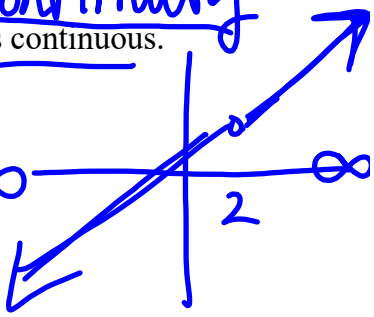
@ $x=2$ the function is undefined

$$\frac{(x+2)(x-2)}{x-2}$$

f(x) has a hole @ $x=2$ and it's a removable discontinuity

State the intervals for which the function is continuous.

$(-\infty, 2) \cup (2, \infty)$




How can we define f(x) so that the function is continuous?

$$f(x)=\frac{x^2-4}{x-2}, x \neq 2$$

① Solve for limit at hole.

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2}$$

$$\lim_{x\to 2} x+2 = 2+2 = 4$$

$$f(x)=\begin{cases} \frac{x^2-4}{x-2}, & x\neq 2 \\ x+2 \text{ ; } 4, & x=2 \end{cases}$$

Is the function continuous? If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

NO

$$f(x)=\frac{1}{x+2}$$

V.A. $x=-2$

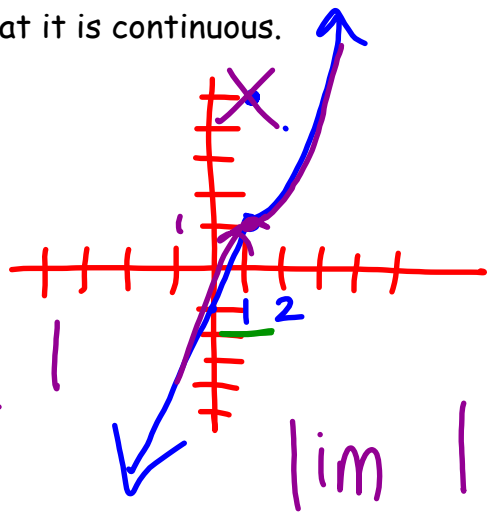
@ $x=-2$

infinite discontinuity @ $x=-2$

State the interval for which the function is continuous.

Rewrite this function so that it is continuous.

$$f(x)=\begin{cases}2x-1, & x<1\\ 5, & x=1\\ x^2, & x>1\end{cases}$$




issue is @ $x=1$

$\lim_{x\to 1^+} 1$

$\lim_{x\to 1^-} 1$

$\lim_{x\to 1} 1$

$f(1)=1$

$$f(x)=\begin{cases}2x-1, & x<1\\ 1 & x=1\\ x^2, & x>1\end{cases}$$

What does a have to be in this function so that it is continuous?

$$f(x)=\begin{cases}2x^2, & x<2\\ ax, & x\geq 2\end{cases}$$

need to make sure that $\lim_{x\to 2}$ exists

$\lim_{x\to 2^-} 2(2)^2 = 8$ ✓

$\lim_{x\to 2^+} f(x) = 8$ ✓

$\lim_{x\to 2^+} ax = 8$

$2(a) = 8$

$\frac{2(a)}{2} = \frac{8}{2}$

$a = 4$

$\lim_{x\to 2} = f(2)$

Is the function continuous? NOT If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

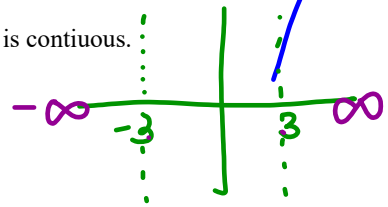
$$f(x)=\frac{x}{x^2-9} \qquad \frac{x}{(x+3)(x-3)}$$

not at $x = \pm 3$

they are infinite discontinuity

State the interval(s) for which the function is contiuous.

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$




Is the function continuous? If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

$y = [\![ x ]\!]$

Greatest Integer Function
The greatest integer less than or equal to *x*.

To put in your calculator
y1 = int (x)

* You can find "int" by pressing 2nd CATALOG (above the 0 key)

HW: TB pg. 84-85
pg. 53-54 packet