Way back in the 17th century. . . .

Isaac Newton

Gottfried Wilhelm Leibniz

1) Tangent Line Problem
   Differential Calculus

2) Area Under Curves
   Integral Calculus

Limits

Intro to Derivatives

Can you solve for the slope between two points?!?!
How?

$(1, 2)$ — x, y

$(4, -3)$ — x, y

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-2}{4-1} = \frac{-5}{3}$$

Now in function notation...

$f(x) =$

$y =$

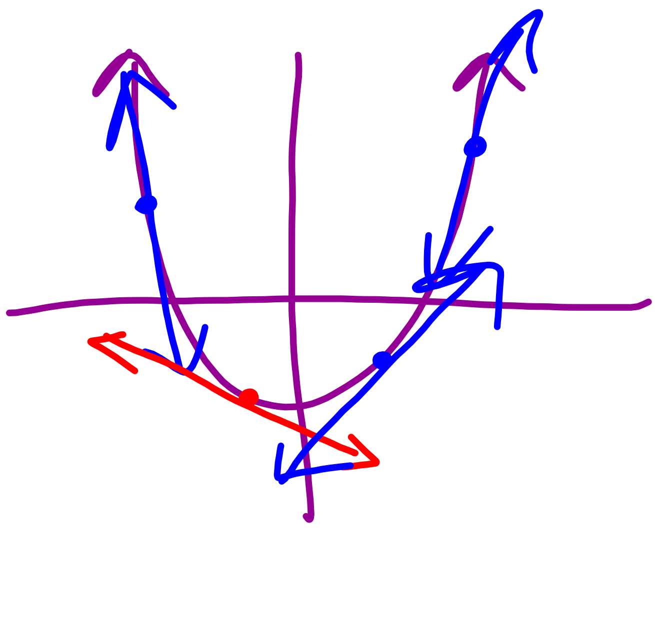
$$slope = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

2.1 The Derivative and the tangent line problem

A derivative is used to find the slope of a tangent line to a curve. In other words, it finds the *instantaneous slope*.

We notate the derivative by $f'(x)$, read "f prime of x."

$f'(x)$



Finding a derivative is called **differentiation**.

The derivative of f(x) can be written different ways:

$$\underline{f'(x)} = \underline{y'} = \underline{y'(x)} = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx} f(x)$$

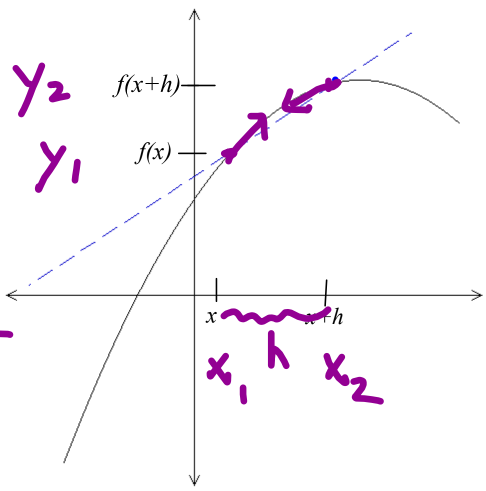
Leibniz Notation

* *dy* and *dx* are called differentials.
It doesn't mean *dy* divided by *dx*.

What is the slope of the line that goes through these two points? What happens as the points get closer to each other?

$$m = \frac{f(x+h) - f(x)}{x+h-x}$$

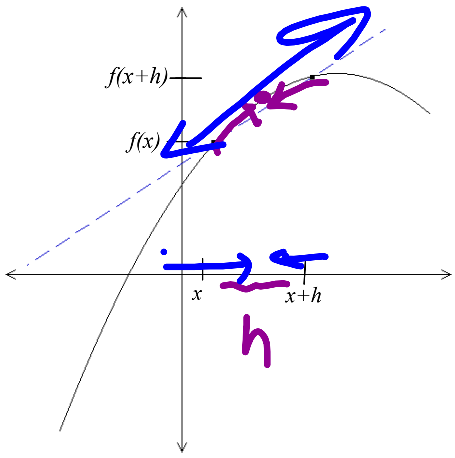
$$m = \frac{f(x+h) - f(x)}{h}$$



Slope of the Tangent Line = Derivative

$$\boxed{m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

This limit is called the "difference quotient."



The Definition of the Derivative

$$f'(x) = m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

or

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

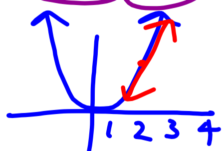
Example 1:

Find the slope on $y = x^2$ at $x = 2$.

$$\underbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$f(x) = x^2$$

$$\boxed{\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}}$$



$$\lim_{h \to 0} \frac{(x+h)(x+h) - x^2}{h} = \frac{\cancel{x^2} + xh + xh + h^2 - \cancel{x^2}}{h}$$

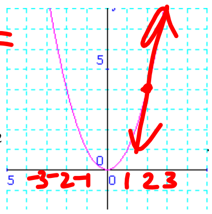
$$\lim_{h \to 0} \frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + 0 = \boxed{2x}$$

$$f'(x) = x^2 = 2x$$

$$f'(2) = 2(2) = 4$$

The slope is: 4

Now look at the graph of $y = x^2$ and check your answer. Does this look like the slope of the tangent line at $x = 2$?



Example 2:

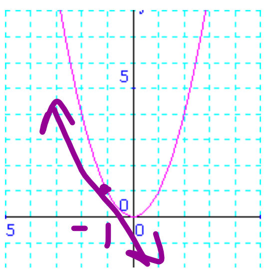
Find the slope on $y = x^2$ at $x = -1$.

$$f'(x) = 2x$$

$$f'(-1) = 2(-1) = -2$$

The slope is: -2

Now look at the graph of $y = x^2$ and check your answer. Does this look like the slope of the tangent line at $x = -1$?



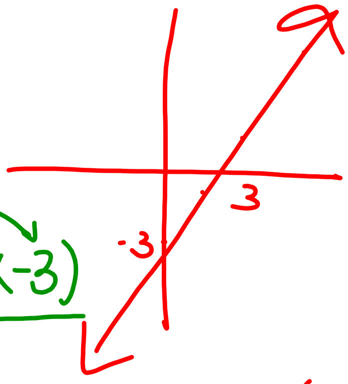
Example 3: $y=mx+b$

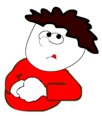
Find the slope on y = $\boxed{2x - 3}$ at x = 3.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{x+h-x}$$

$$\lim_{h\to 0} \frac{2(x+h)-3-(2x-3)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{2x}+2h-\cancel{3}-\cancel{2x}+\cancel{3}}{h} = \frac{2\cancel{h}}{\cancel{h}} = 2$$





How could we have done this in under 2 seconds flat?!?!

Example 4:

Write the equation of a line tangent to $y = -2x^2$ at $x = 1$

What two things do we need to write the equation of a line?

1.

2.

Use the definition of a derivative:

Find $f'(x)$ if $f(x) = 3x^2 - 5x$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}$$

$$\lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 5x - 5h - 3x^2 + 5x}{h}$$

$$\lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{5x} - 5h - \cancel{3x^2} + \cancel{5x}}{h}$$

$$\lim_{h\to 0} \frac{6xh + 3h^2 - 5h}{h} = \frac{\cancel{h}(6x+3h-5)}{\cancel{h}}$$

$$\lim_{h\to 0} 6x + 3(0) - 5 = \boxed{6x - 5}$$

$f(x) = 3x^2 - 5x^1$

$f'(x) = 6x - 5$

$f(x) = x^{2-1}$

$f'(x) = 2x^1$

Find $f'(x)$.

$f(x) = 7$

$\cancel{f''(x)}$

$f(x) = 2x^3 + 7x^0$

$f'(x) = 3 \cdot 2x^{3-2}$

$\boxed{= 6x^2}$

Find $\frac{dy}{dx}$.

$$y = \sqrt{x+2}$$

$f(x) = 3x^2 - 5x$

Find the ***EQUATION Of THE TANGENT LINE***
at $x = -1$.

$$y = 3/x$$

Find the equation for the tangent line at (1,3).

Attachments

Graph 2.1 Limit Tangent Line.tii

Define Derivative & NDER.gsp