Limits at Infinity

Asking what happens as x gets very large ($x \to \infty$)

Or what happens as x gets very small ($x \to -\infty$)

Essentially...

Limits as x approaches infinity = **End Behavior**

Example 1A: Find $\lim_{x\to\infty} \frac{x}{x^2+1}$ using a table.

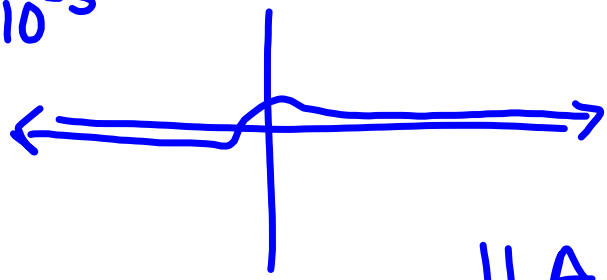
H.A.

bottom heavy H.A. $y=0$

x	f(x)
1	.5
10	.09901
100	.01
1000	1E-3
10000	.0001
100000	.00001
1000000	

.001 1×10^{-3}

Example 1B: Find $\lim_{x\to\infty} \frac{x}{x^2+1}$ using a graph.



1) bottom heavy H.A. $y=0$

2) top heavy no H.A. $\pm\infty$

3) equal H.A. $y=\frac{LC}{LC}$

Example 1C: Find $\lim_{x\to\infty} \frac{x}{x^2+1}$ using algebra!

The line y = 0 is the horizontal asymptote of the graph, thus it is the limit as x approaches infinity!!

But, how do we find the horizontal asymptote for any graph???

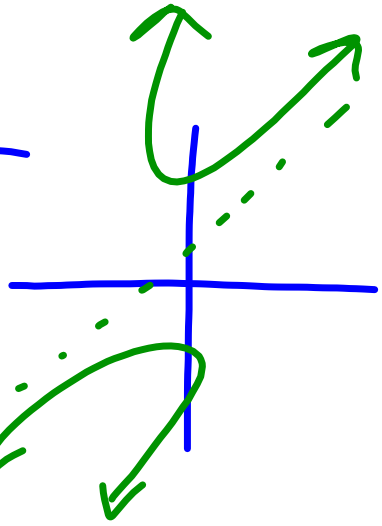
$$\frac{x}{x^2+1} = \frac{x}{x^2} = \frac{1}{x}$$

1. If the degree of numerator > degree of denominator, then there is no horizontal asymptote.

ex: $f(x)=\frac{x^2+2}{x+1}$ $\frac{x^2}{x}=\frac{x}{1}$

$\lim_{x\to+\infty} x=+\infty$

$\lim_{x\to-\infty} x=-\infty$



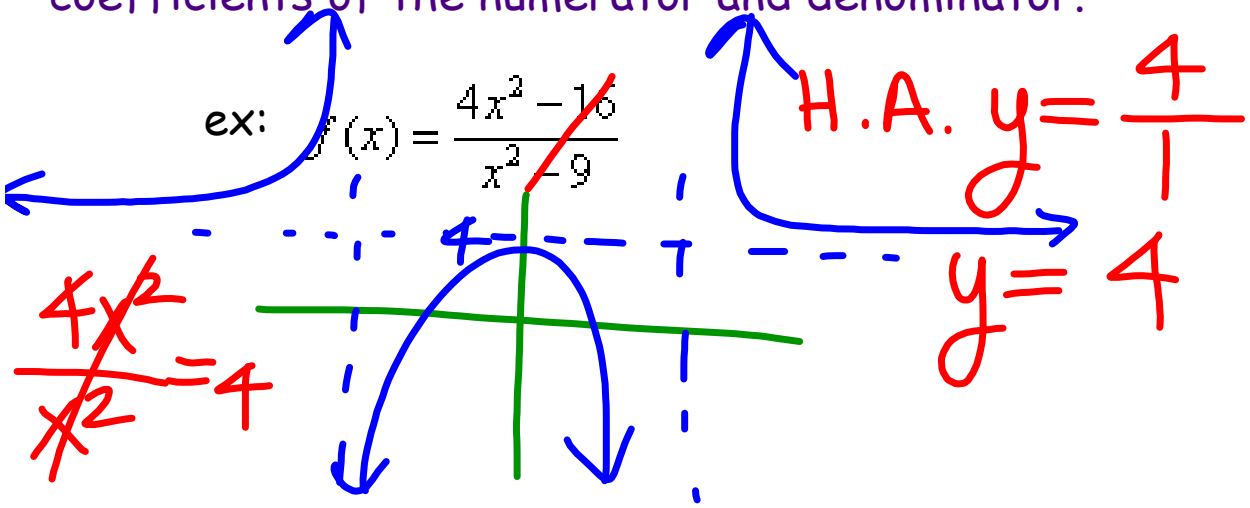
2. If the degree of numerator < degree of denominator, then there is a horizontal asymptote at y=0

ex: $f(x) = \frac{x}{x^2 - 4} = \frac{x}{x^2} = \frac{1}{x}$ H.A. $y = 0$

$\lim_{x \to +\infty} \frac{1}{x} = 0$

3. If the degree of numerator = degree of denominator, there is a HA at y = a/b, where a and b are leading coefficients of the numerator and denominator.

ex: $f(x) = \frac{4x^2 - 16}{x^2 - 9}$



Examples!

$$\lim_{x\to\infty} \frac{2-3x+x^2}{7+4x-5x^2} = -\frac{1}{5}$$

H.A. $y = -\frac{1}{5}$

$$\frac{x^2}{-5x^2} = \frac{1}{-5}$$

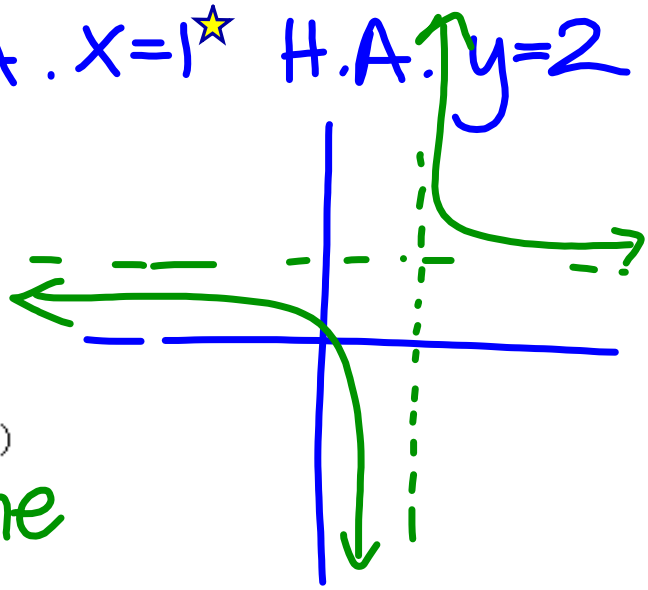
$$\lim_{x\to\infty} \frac{2x^3}{1+x^3} = 2$$ b/c H.A. $y=2$

Put it all together:

$$f(x)=\frac{2x}{x-1}$$

1. Find the VA and HA V.A. $x=1$ H.A. $y=2$

2. Find: $\lim_{x\to\infty} f(x)$ $=2$



3. Find: $\lim_{x\to 1+} f(x)$ $\lim_{x\to 1-} f(x)$ $\lim_{x\to 1} f(x)$

$+\infty$ $-\infty$ dne