

A function that is a ratio of two polynomials.

Standard Form: $f(x) = \frac{x^2 - 5x + 6}{x^3 - 1}$

Transformation Form: $g(x) = \frac{2}{x-1} - 3$

RATIONAL FUNCTIONS:

Vertical Asymptote

Any value of x that makes denominator equal to zero.
 *** Apart from a factor that cancels out (that's a hole)****. Factor denominator (bottom) and set factor(s) equal to zero and solve. Answers should be written as a line $x =$

*The graph will NEVER cross the V.A.

Y-intercept

Plug zero in for x in the original equations. Confirm with table on calculator. Answer should be an ordered pair $(0, \quad)$

The point where the graph crosses the y -axis

X-intercept

Set the numerator (top) equal to zero and solve. *** After cancelling holes*** Confirm with table on calculator. Answers should be an ordered pair $(\quad, 0)$

The point where the graph crosses the x -axis

Domain

All values of " x " excluding x values of the holes and the V.A.

Domain: $\{x \mid x \neq \quad\}$

*

Can't use x -values that make denominator = 0.

Horizontal Asymptote

Look at the degree of the numerator and denominator.

1) Degree of denominator > degree of numerator $\rightarrow y = 0$ (x -axis) - bottom heavy

2) Degree of numerator = degree of denominator $\rightarrow y =$ leading coefficient of numerator / leading coefficient of denominator

H.A. $y = \frac{0}{0}$

H.A. $y = \frac{16}{16}$

3) Degree of numerator > degree of denominator \rightarrow There is no horizontal asymptote. -top heavy

No H.A., there may be a slant asymptote.

Slant Asymptote

This occurs when the degree of the numerator is exactly one more than the degree of denominator.

There are exceptions when the graph crosses the H.A.

All values of " y " excluding y values of the holes and H.A.

Range: $\{y \mid y \neq \quad\}$

Range

Hole

Factor the numerator (top), factor the denominator (bottom). If they have a common factor that cancels, this is your hole. Set this factor equal to zero and solve. Plug that " x " value back into simplified FF to find " y " coordinate of the hole. Answer should be written as an ordered pair. (x,y)

Ex. $\frac{(x+1)(x-2)}{(x+1)(x+5)}$ hole $(-1, -1)$

This is controlled by the H.A. It is read as:

$x \rightarrow \infty, y \rightarrow \frac{H.A.}{H.A.}$

$x \rightarrow -\infty, y \rightarrow \frac{H.A.}{H.A.}$

End Behavior

