Calculus

Unit 1

Limits and Continuity

Name: ____________________

Date: ___________ Block: ____

Calculus Review - Polynomials

WS 2: Mixed Factoring Practice

MM3A3: Students will solve a variety of equations and inequalities.

Factor completely.

1. x^2+5x+6 2. x^2-64 3. x^3+125
4. $6x^2-5x-6$ 5. $x^3+6x^2-5x-30$ 6. $-63ab^2+18b^2-27ab^3$
7. $x^5-2x^4-9x+18$ 8. $9x^2+52x-77$ 9. x^3-5x^2-6x
10. x^3+4x^2+2x+8 11. $4x^2-25x^6$ 12. $35cb^2-14cb-21c$
13. a^3b^3-64 14. $x^3-4x+5x^2-20$ 15. $x^2+2x-xy-2y$
16. $2c^3+16$ 17. $27x^4-75$ 18. x^3-16x^2+64x
19. $8x^6-125$ 20. $3a^2+14a+15$ 21. x^8-81
22. $2x^2+3y-xy-6x$ 23. $8x^3-27y^3$ 24. $56def-21cea+35aeg-14bef$

GPS Calculus

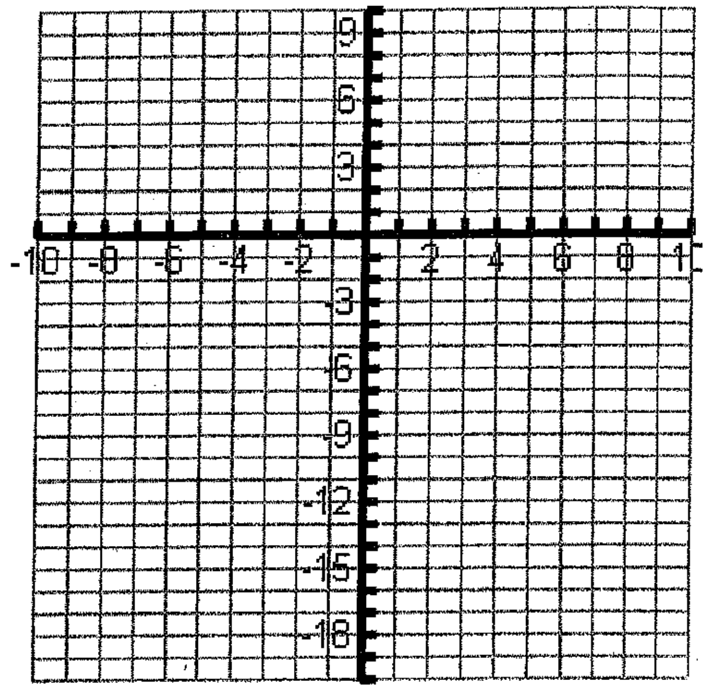
Name:____________________

Graphs and Characteristics of Polynomials

Graph Scales GIVEN

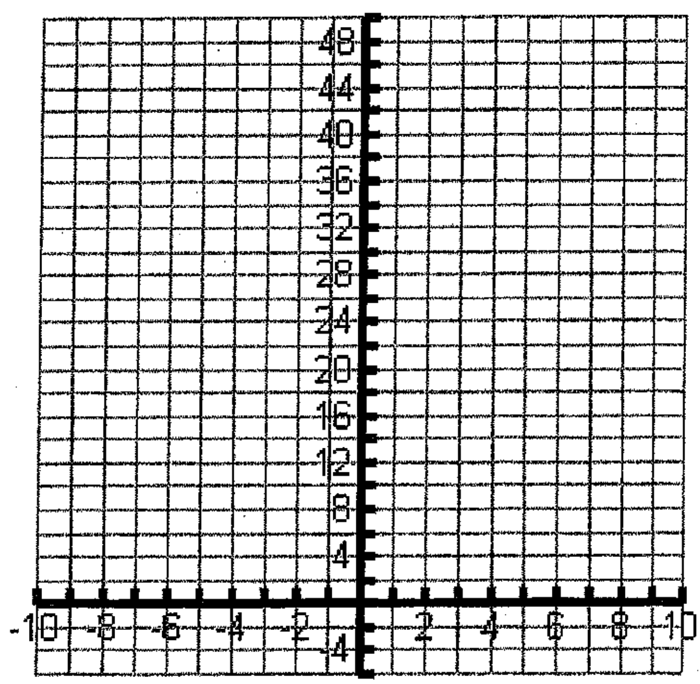
Characteristics of Polynomials – Describe all the characteristics of each polynomial. Sketch each of the graphs

$f(x) = x^3 - 5x^2 - x + 5$



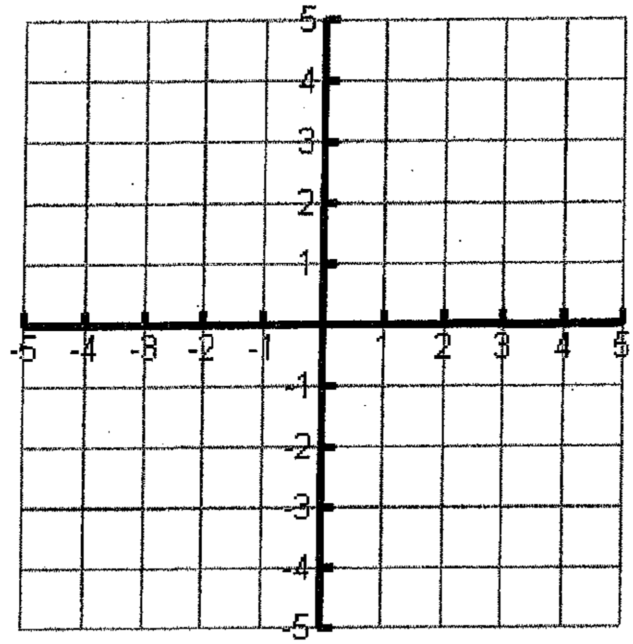
1. Domain:______________
   Range:______________
   Zeros:____________________
   x-int:____________________
   y-int:______________
   end behavior:
   ____
   Maximums: global:______________
   Local: ____________________
   Minimums: global: ______________
   Local: ____________________
   Intervals: increasing: ____________________
   decreasing: ____________________

$f(x) = x^4 - 13x^2 + 40$



2. Domain:______________
   Range:______________
   Zeros:____________________
   x-int:____________________
   y-int:______________
   end behavior:
   ____
   Maximums: global:______________
   Local: ____________________
   Minimums: global: ______________
   Local: ____________________
   Intervals: increasing: ____________________
   decreasing: ____________________

$f(x) = 4x^3 - x^2 - 4x + 1$



3. Domain:_______________

Range:_______________

Zeros:_______________________

x-int:_______________________

y-int:_______________

end behavior:

____

Maximums: global:______________

Local: _______________________

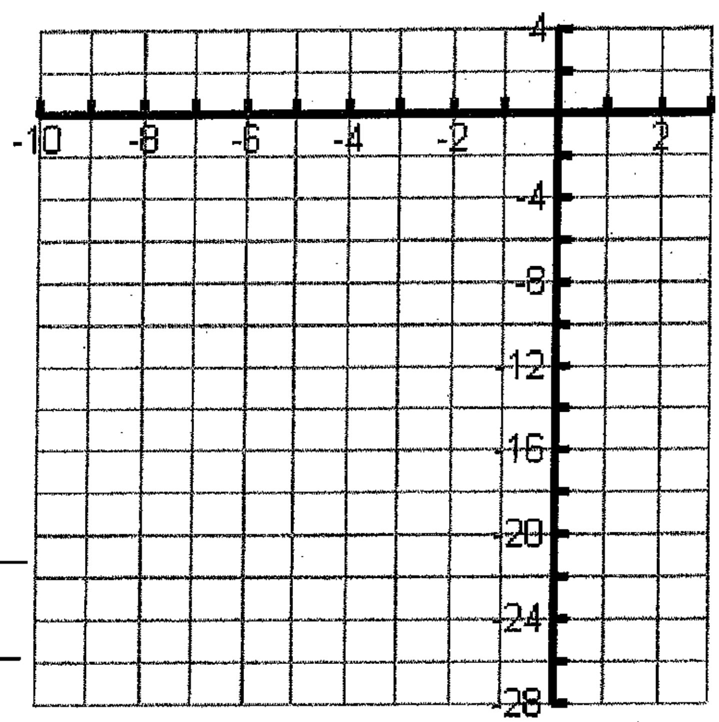
Minimums: global: ______________

Local: _______________________

Intervals: increasing: __________________________

decreasing: __________________________

$f(x) = 5x^2 + 29x + 20$



Domain:_______________

4.

Range:_______________

Zeros:_______________________

x-int:_______________________

y-int:_______________

end behavior:

____

Maximums: global:______________

Local: _______________________

Minimums: global: ______________

Local: _______________________

Intervals: increasing: __________________________

decreasing: __________________________

Standards: MM4A1, MM4A4

I. For the following problems, match the function with its graph.

1. $f(x)=\dfrac{2}{x+2}$

2. $f(x)=\dfrac{1}{x-3}$

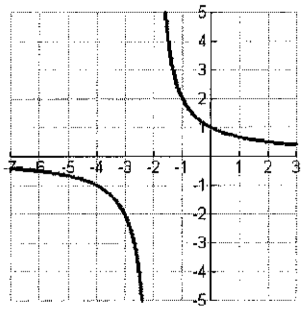
3. $f(x)=\dfrac{4x+1}{x}$

4. $f(x)=\dfrac{1-x}{x}$

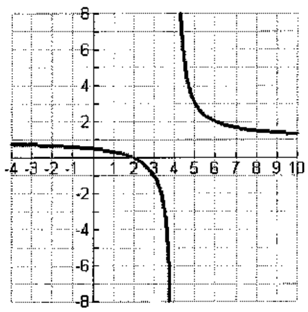
5. $f(x)=\dfrac{x-2}{x-4}$

6. $f(x)=-\dfrac{x+2}{x+4}$

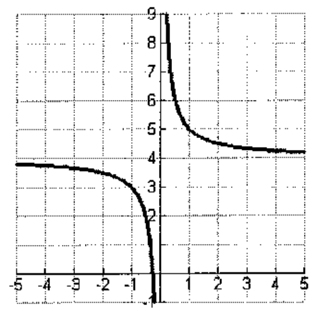
a)



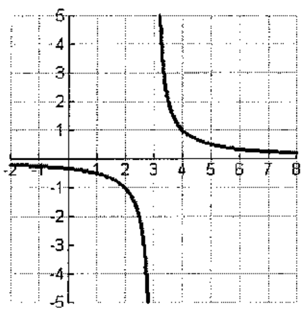
b)



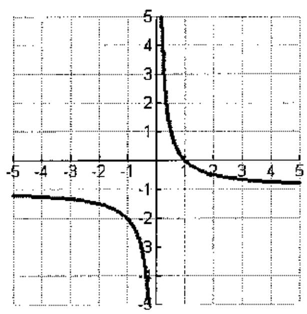
c)



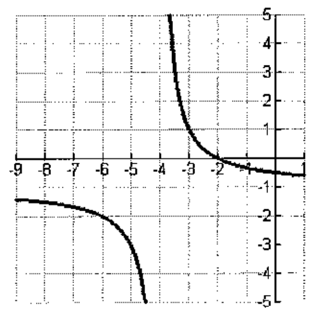
d)



e)

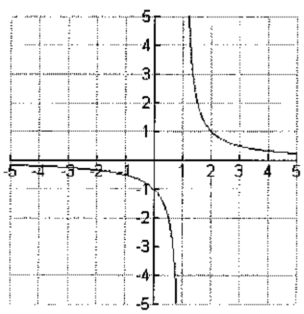


f)

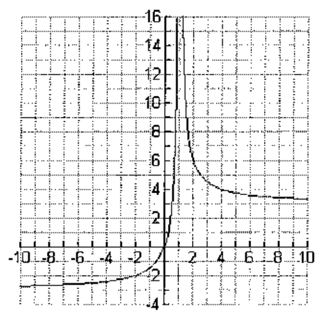


II. For the following problems, a) determine the vertical, horizontal and slant asymptotes of the function, b) state any points of discontinuity, and c) find the domain and range of the function. Do not use a graphing calculator except to check your answers.

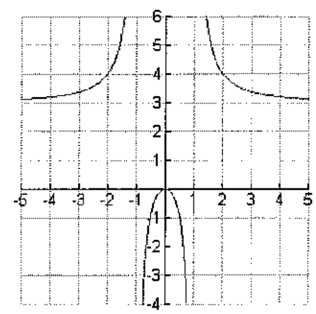
7. $f(x) = \dfrac{1}{x-1}$



8. $f(x) = \dfrac{3x}{|x-1|}$



9. $f(x) = \dfrac{3x^2}{x^2-1}$



What is a Limit???

http://archives.math.utk.edu/visual.calculus/1/limits.16/tut1-flash.html

Intuitive Definition. Let **y = f(x)** be a function. Suppose that **a** and **L** are numbers such that:

- whenever **x** is close to **a** but not equal to **a**, **f(x)** is close to **L**;
- as **x** gets closer and closer to **a** but not equal to **a**, **f(x)** gets closer and closer to **L**; and
- suppose that **f(x)** can be made as close as we want to **L** by making **x** close to **a** but not
  equal to **a**.

Then we say that **the limit of f(x) as x approaches a is L** and we write

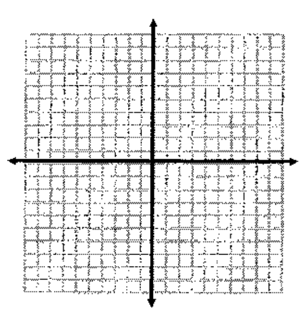
$$\lim_{x \to c} f(x) = L$$

Evaluating a Limit Graphically and Numerically

a) Numerically $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2}$

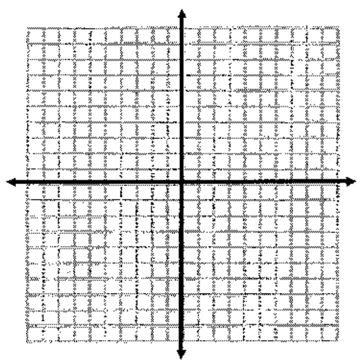
x	
$g(x)$	

b) Graphically



$$\lim_{x \to 2} \frac{x|x-2|}{x-2}$$

x	
g(x)	



Limits DO NOT EXIST when there is:

1. Behavior that differs from left to right...

$$\lim_{x \to 1} \frac{|x-1|}{x-1}$$

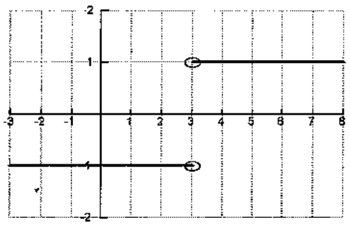
2. Unbounded Behavior

$$\lim_{x \to 0} \frac{2}{x^2}$$

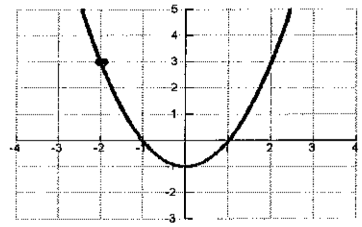
3. Oscillating Behavior

$$\lim_{x \to 0} sin\frac{1}{x}$$

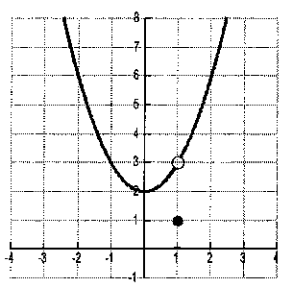
Use the graph to find the limit if it exists.



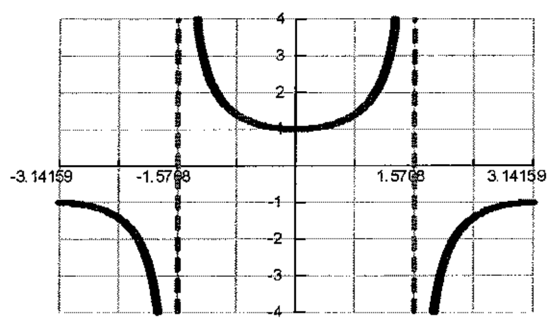
1. $\lim_{x \to 3} f(x)$



2. $\lim_{x \to -2} f(x)$



3. $\lim_{x \to 1} f(x)$



4. $\lim_{x \to 0} f(x)$

One sided limits

$$\lim_{x \to a^-} f(x) = L$$

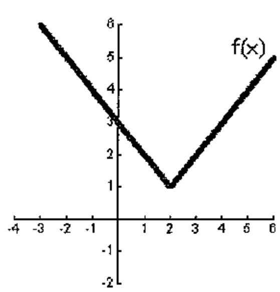
"The limit of f(x) as x approaches a from the left."

$$\lim_{x \to a^+} f(x) = L$$

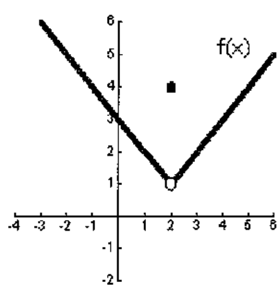
"The limit of f(x) as x approaches a from the right."

Find $\lim_{x \to 2} (f(x))$

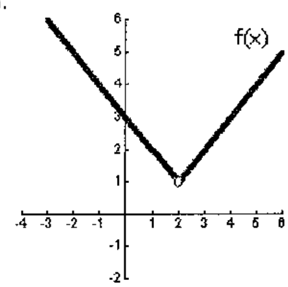
1.



2.



3.



$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

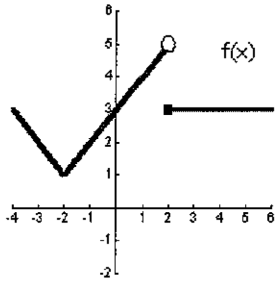
$\lim_{x \to 2} f(x) =$

$\lim_{x \to 2^+} f(x) =$

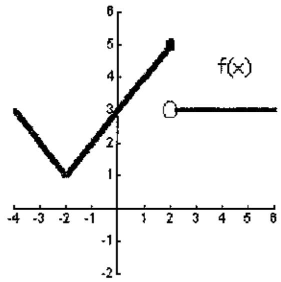
$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

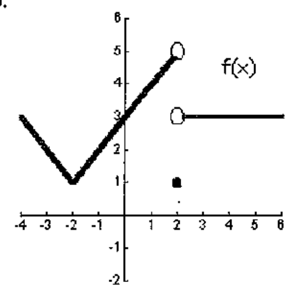
4.



5.



6.



$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

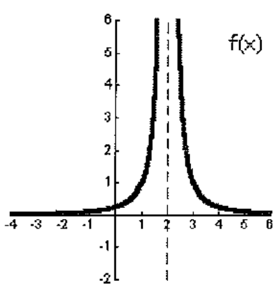
$\lim_{x \to 2} f(x) =$

$\lim_{x \to 2^+} f(x) =$

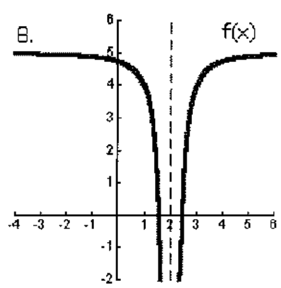
$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

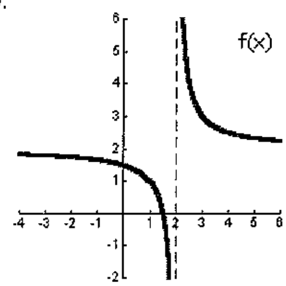
7.



8.



9.



$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

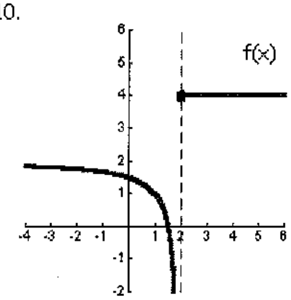
$\lim_{x \to 2} f(x) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

10.

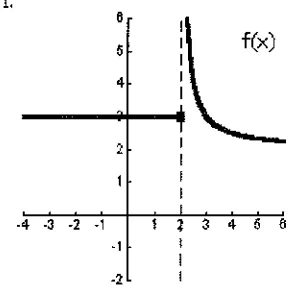


$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

11.

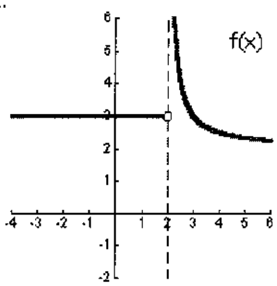


$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

12.

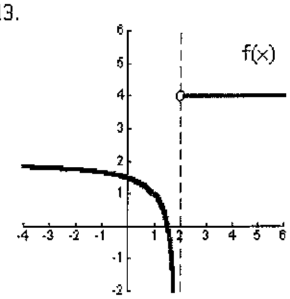


$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

13.



$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

Example

Evaluate the limits graphically then numerically.

a) Graphically.

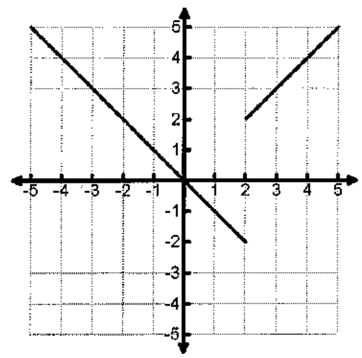
$$g(x) = \frac{x|x-2|}{x-2}$$

$\lim_{x \to 2^+} g(x) =$

$\lim_{x \to 2^-} g(x) =$

$\lim_{x \to -3^-} g(x) =$

$\lim_{x \to -3^+} g(x) =$



b) Numerically.

$$g(x) = \frac{x|x-2|}{x-2}$$

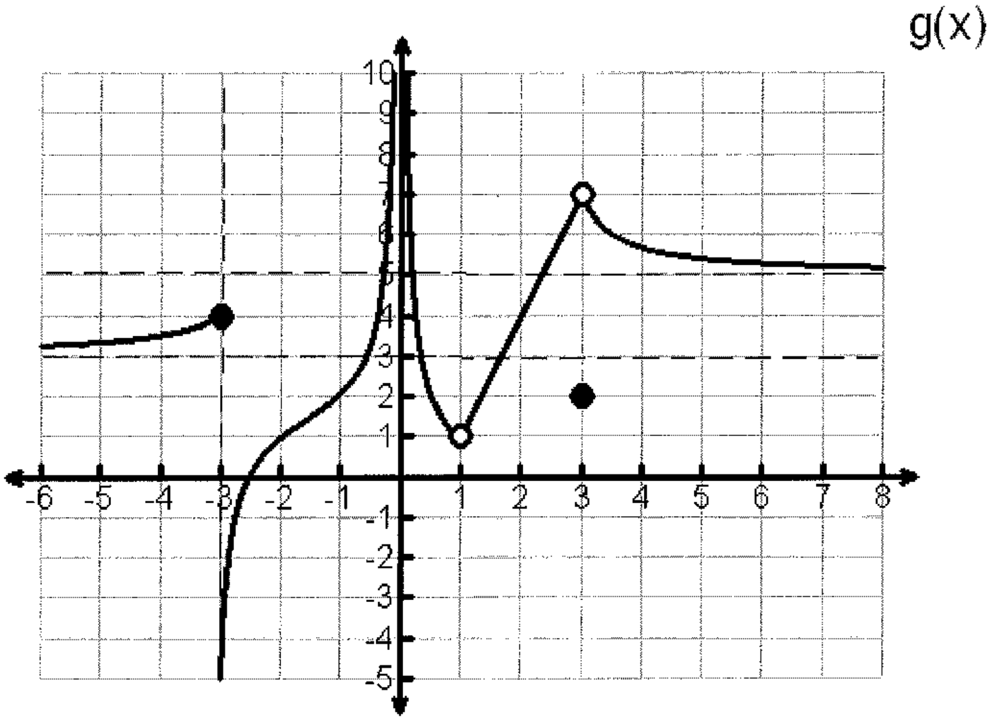
$\lim_{x \to 2^+} g(x) =$ $\lim_{x \to 2^-} g(x) =$

x	
g(x)	

$\lim_{x \to -3^-} g(x) =$ $\lim_{x \to -3^+} g(x) =$

x	
g(x)	

Limits Graphically Worksheet
Honors Calculus



Evaluate each of the following.

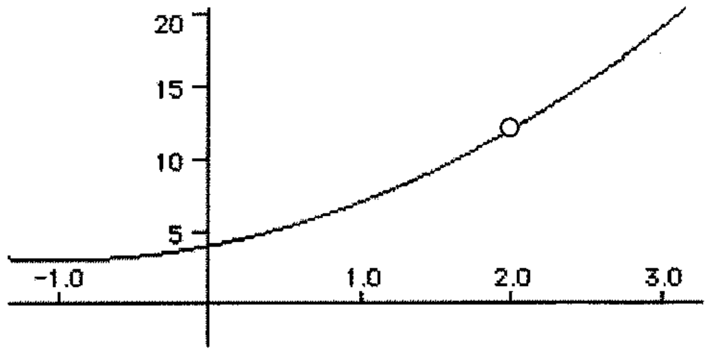
1. $\lim_{x \to -3^+} g(x)$
2. $\lim_{x \to -3^-} g(x)$
3. $\lim_{x \to -3} g(x)$
4. $g(-3)$
5. $\lim_{x \to -1^+} g(x)$
6. $\lim_{x \to -1} g(x)$
7. $g(-1)$
8. $g(0)$
9. $\lim_{x \to 0} g(x)$
10. $\lim_{x \to 1^+} g(x)$
11. $\lim_{x \to 1^-} g(x)$
12. $\lim_{x \to 1} g(x)$
13. $g(1)$
14. $\lim_{x \to 3} g(x)$
15. $g(3)$
16. $\lim_{x \to -\infty} g(x)$
17. $\lim_{x \to \infty} g(x)$

Graphical Approach to Limits - Classwork

Suppose you were to graph

$f(x)=\dfrac{x^3-8}{x-2},\ x\neq 2$

For all values of x not equal to 2, you can use standard curve sketching techniques. But the curve is not defined at $x=2$. There is a hole in the graph. So let's get an idea of the behavior of the curve around $x=2$.



Set your calculator to 4 decimal accuracy and complete the chart.

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$f(x)$									

It should be obvious that as x gets closer and closer to 2, the value of $f(x)$ becomes closer and closer to ________.

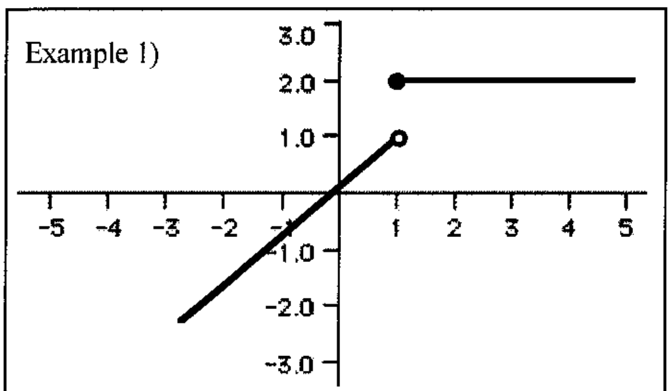
We will say that the **limit** of $f(x)$ as x approaches 2 is 12 and this is written as $\lim\limits_{x\to 2} f(x)=12$ or $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}=12$.

The informal of definition of a limit is "what is happening to y as x gets close to a certain number." In order for a limit to exist, we must be approaching the same y-value as we approach some value c from either the left or the right side. If this does not happen, we say that the limit does not exist (DNE) as we approach c.

If we want the limit of $f(x)$ as we approach some value of c from the left hand side, we will write $\lim\limits_{x\to c^-} f(x)$.

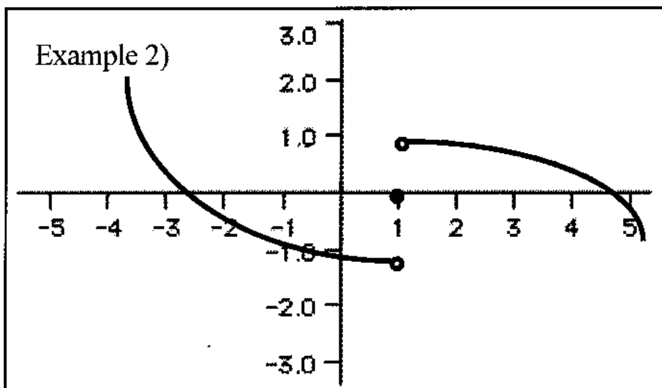
If we want the limit of $f(x)$ as we approach some value of c from the right hand side, we will write $\lim\limits_{x\to c^+} f(x)$.

In order for a limit to exist at c, $\lim\limits_{x\to c^-} f(x)$ must equal $\lim\limits_{x\to c^+} f(x)$ and we say $\lim\limits_{x\to c} f(x)=L$.



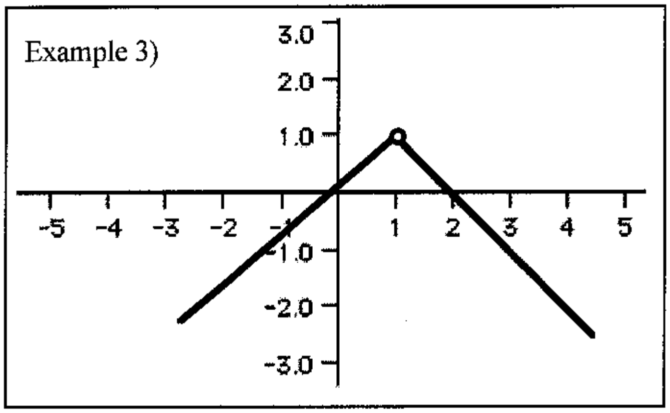
$\lim\limits_{x\to 1^-} f(x)=$ ______ $\lim\limits_{x\to 1^+} f(x)=$ ______

$\lim\limits_{x\to 1} f(x)=$ ______ $f(1)=$ ______



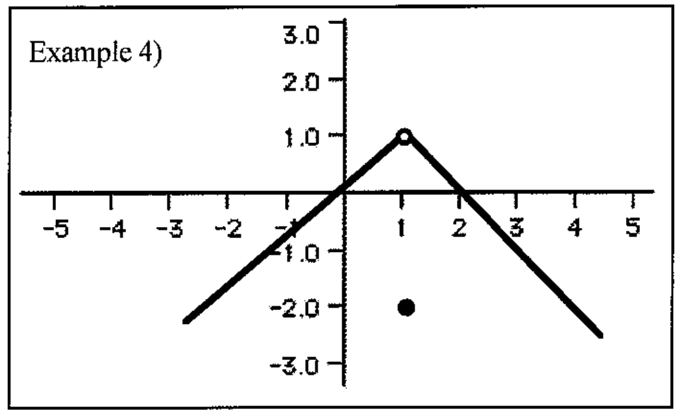
$\lim\limits_{x\to 1^-} f(x)=$ ______ $\lim\limits_{x\to 1^+} f(x)=$ ______

$\lim\limits_{x\to 1} f(x)=$ ______ $f(1)=$ ______



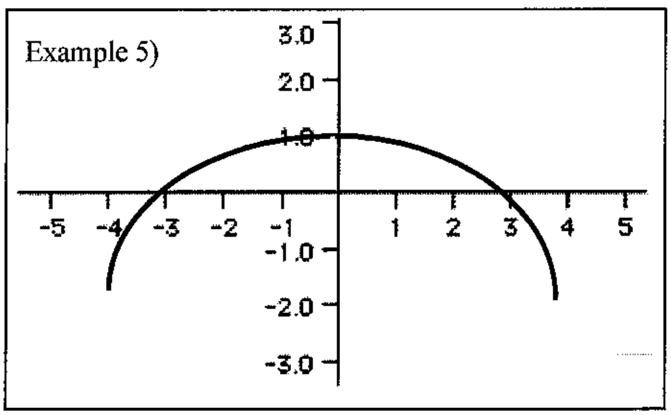
$\lim_{x \to 1^-} f(x) =$ ______ $\lim_{x \to 1^+} f(x) =$ ______

$\lim_{x \to 1} f(x) =$ ______ $f(1) =$ ______



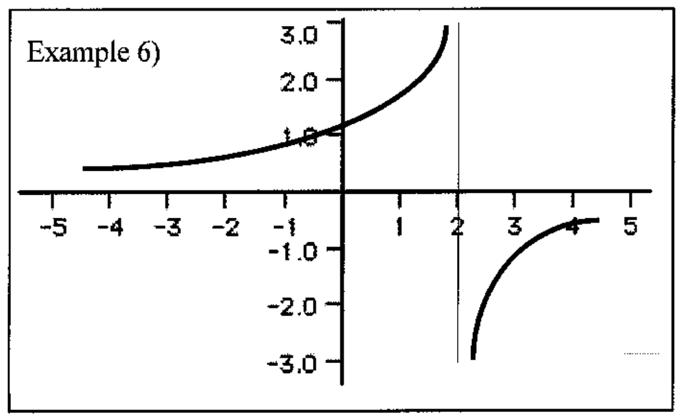
$\lim_{x \to 1^-} f(x) =$ ______ $\lim_{x \to 1^+} f(x) =$ ______

$\lim_{x \to 1} f(x) =$ ______ $f(1) =$ ______



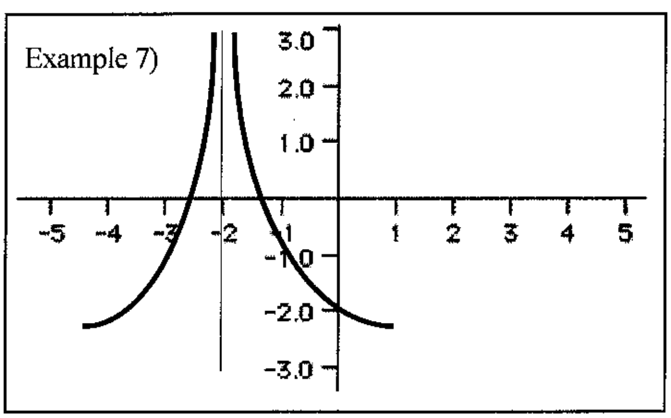
$\lim_{x \to 0^-} f(x) =$ ______ $\lim_{x \to 0^+} f(x) =$ ______

$\lim_{x \to 0} f(x) =$ ______ $f(0) =$ ______



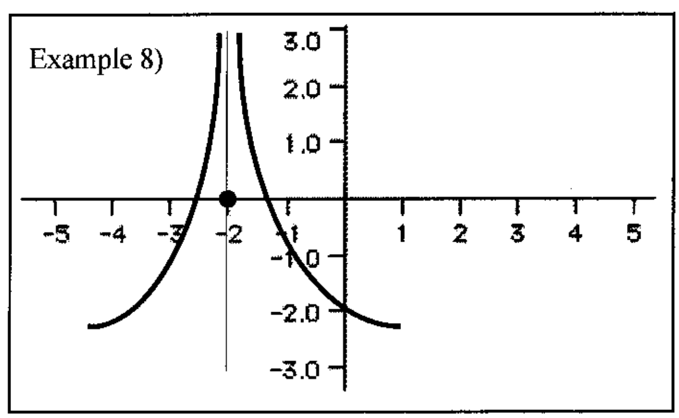
$\lim_{x \to 2^-} f(x) =$ ______ $\lim_{x \to 2^+} f(x) =$ ______

$\lim_{x \to 2} f(x) =$ ______ $f(2) =$ ______



$\lim_{x \to -2^-} f(x) =$ ______ $\lim_{x \to -2^+} f(x) =$ ______

$\lim_{x \to -2} f(x) =$ ______ $f(-2) =$ ______



$\lim_{x \to -2^-} f(x) =$ ______ $\lim_{x \to -2^+} f(x) =$ ______

$\lim_{x \to -2} f(x) =$ ______ $f(-2) =$ ______

The concept of limits as x approaches infinity means the following: "what happens to y as x gets infinitely large." We are interested in what is happening to the y-value as the curve gets farther and farther to the right. We can also talk about limits as x approaches negative infinity. This means what is happening to the y-value as the curve gets farther and farther to the left. The terminology we use are the following: $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

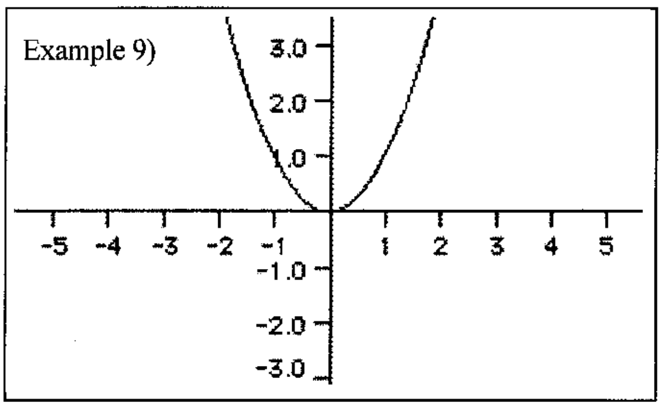
Although we use the term "as x approaches infinity", realize that x cannot approach infinity as infinity does not exist. The term "x approaches infinity" is just a convenient way to talk about the curve infinitely far to the right.

Note that it makes no sense to talk about $\lim_{x\to\infty^+} f(x)$ or $\lim_{x\to-\infty^-} f(x)$. Why? ________________________________

There are only 4 possibilities for $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$:

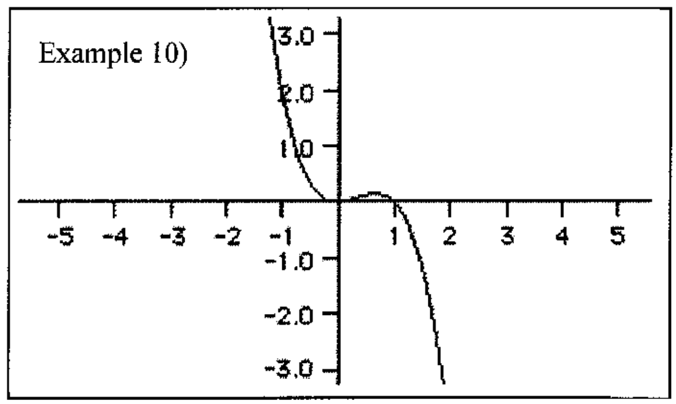
- the curve can go up forever. In that case, the limit does not exist. For convenience sake, we will say $\lim_{x\to\infty} f(x) = \infty$
- the curve can go down forever. In that case, the limit does not exist. For convenience sake we will say $\lim_{x\to\infty} f(x) = -\infty$

Example 9)



In this case, $\lim_{x\to-\infty} f(x) =$ ______________

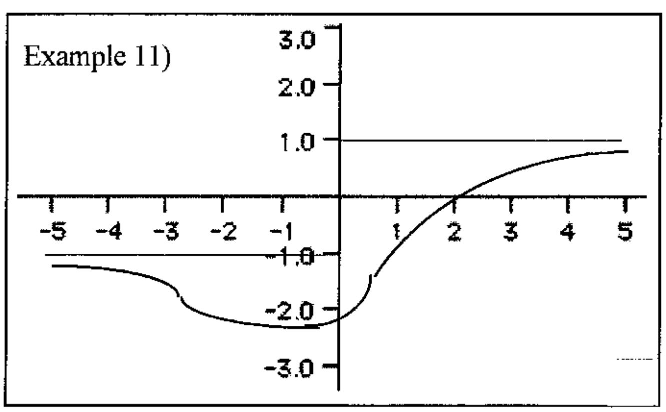
Example 10)



In this case, $\lim_{x\to-\infty} f(x) =$ ______________

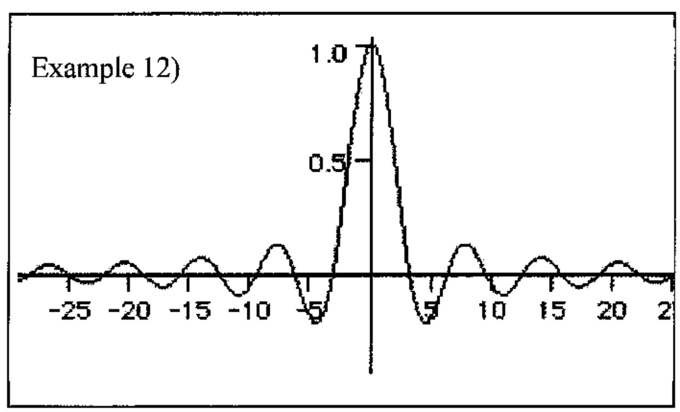
- the curve can become asymptotic to a line. In that case the limit as x approaches infinity is a value.
- the curve can level off to a line. In that case, the limit as x approaches infinity is a value.

Example 11)



In this case, $\lim_{x\to\infty} f(x) =$ _____ and $\lim_{x\to-\infty} f(x) =$ _____

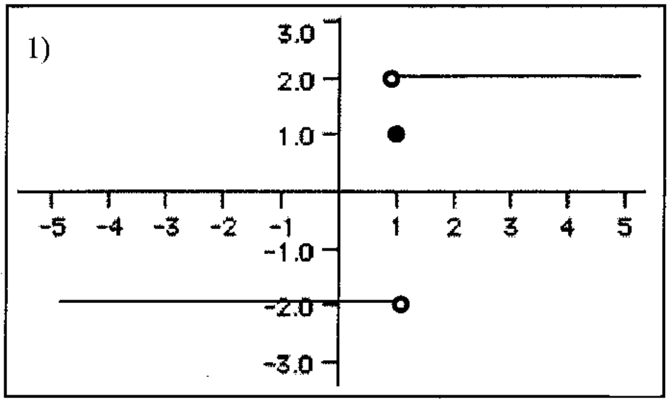
Example 12)



In this case, $\lim_{x\to\infty} f(x) =$ _____ and $\lim_{x\to-\infty} f(x) =$ _____

Graphical Approach to Limits - Homework

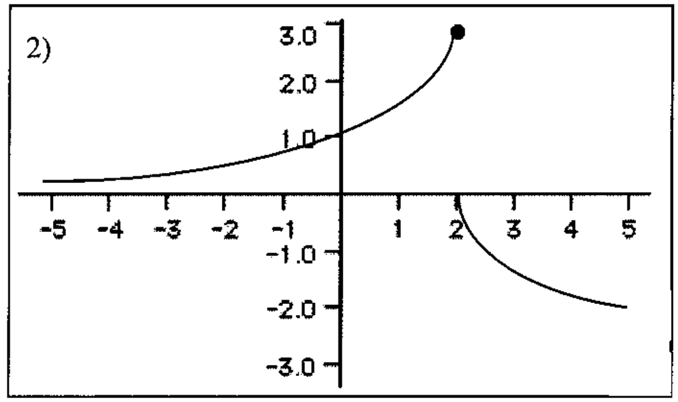
1)



a) $\lim_{x\to 1^-} f(x)$ b) $\lim_{x\to 1^+} f(x)$ c) $\lim_{x\to 1} f(x)$

d) $f(1)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

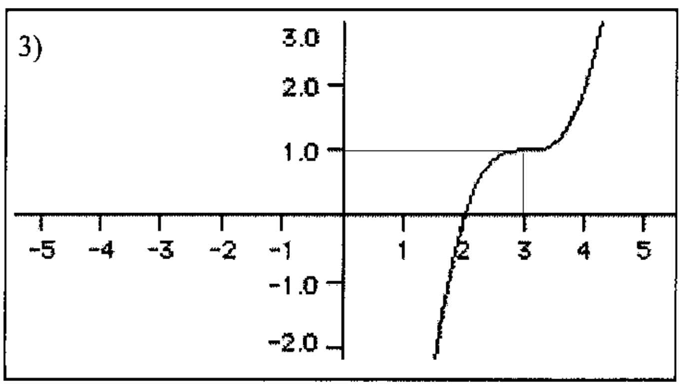
2)



a) $\lim_{x\to 2^-} f(x)$ b) $\lim_{x\to 2^+} f(x)$ c) $\lim_{x\to 2} f(x)$

d) $f(2)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

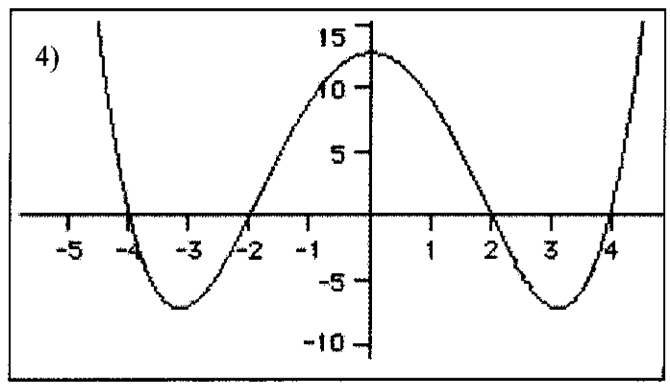
3)



a) $\lim_{x\to 3^-} f(x)$ b) $\lim_{x\to 3^+} f(x)$ c) $\lim_{x\to 3} f(x)$

d) $f(3)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

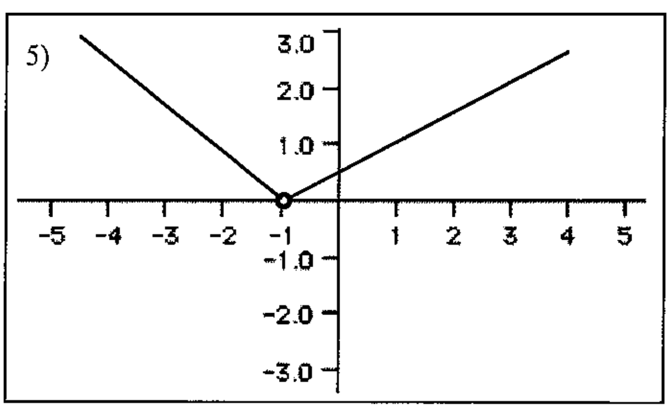
4)



a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$

d) $f(0)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

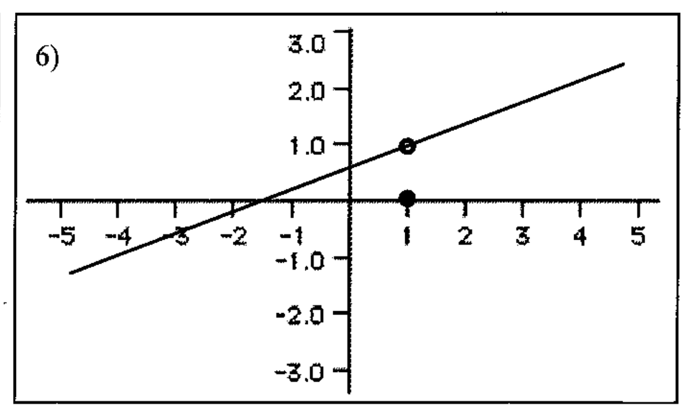
5)



a) $\lim_{x\to -1^-} f(x)$ b) $\lim_{x\to -1^+} f(x)$ c) $\lim_{x\to -1} f(x)$

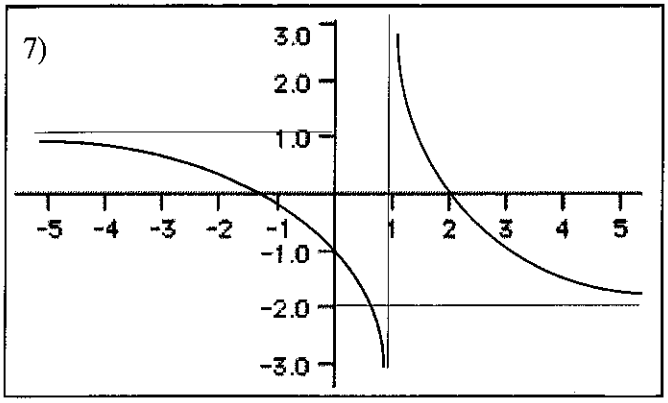
d) $f(-1)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

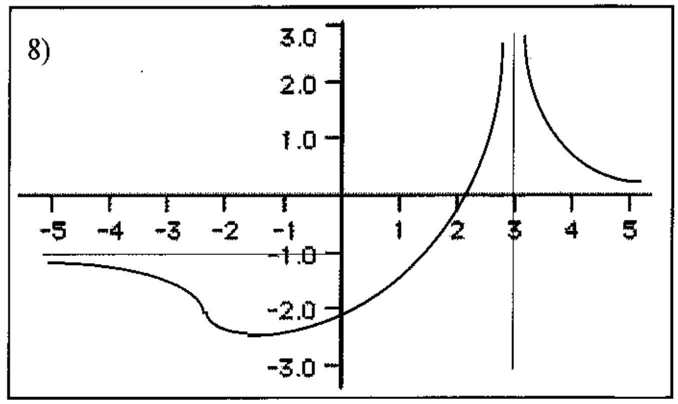
6)



a) $\lim_{x\to 1^-} f(x)$ b) $\lim_{x\to 1^+} f(x)$ c) $\lim_{x\to 1} f(x)$

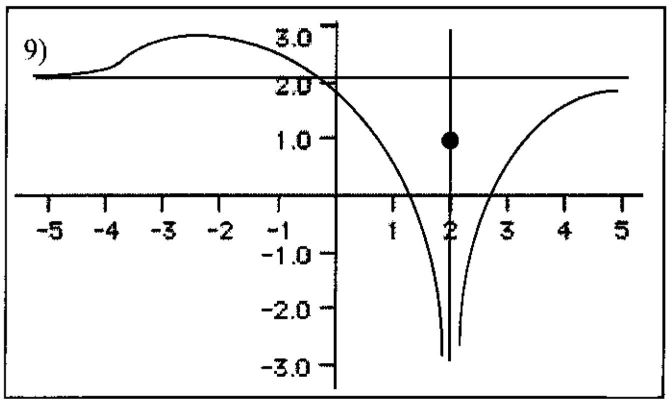
d) $f(1)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

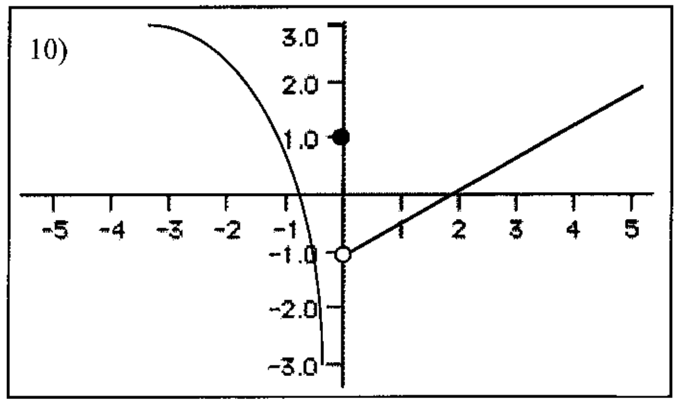




a) $\lim_{x\to 1^-} f(x)$ b) $\lim_{x\to 1^+} f(x)$ c) $\lim_{x\to 1} f(x)$
d) $f(1)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

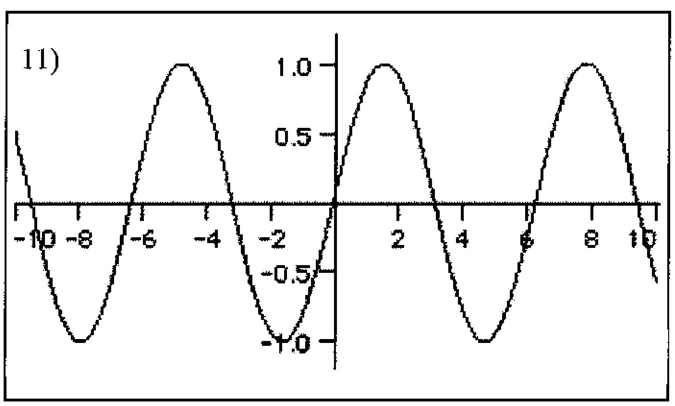
a) $\lim_{x\to 3^-} f(x)$ b) $\lim_{x\to 3^+} f(x)$ c) $\lim_{x\to 3} f(x)$
d) $f(3)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

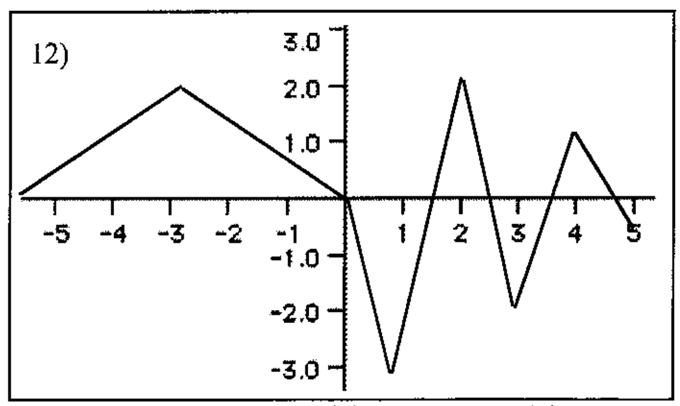




a) $\lim_{x\to 2^-} f(x)$ b) $\lim_{x\to 2^+} f(x)$ c) $\lim_{x\to 2} f(x)$
d) $f(2)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$
d) $f(0)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

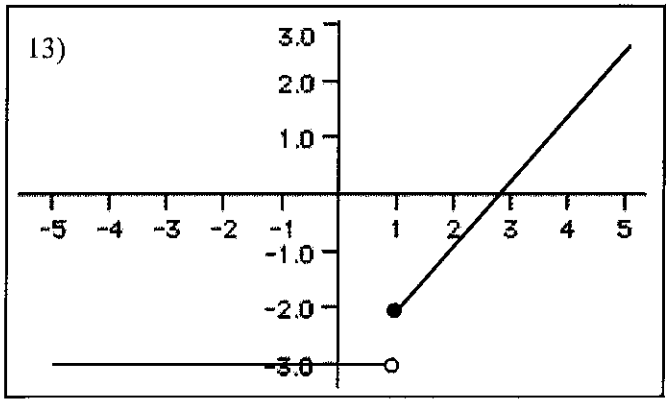




a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$
d) $f(0)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$
d) $f(0)$ e) $\lim_{x\to -\infty} f(x)$ f) $\lim_{x\to \infty} f(x)$

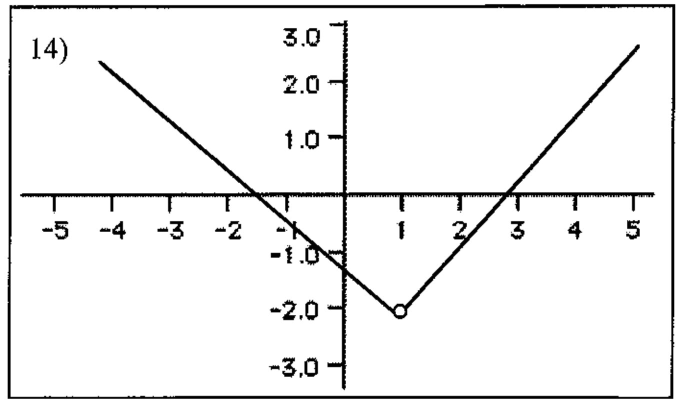
13)



a) $\lim\limits_{x\to 1^-} f(x)$ b) $\lim\limits_{x\to 1^+} f(x)$ c) $\lim\limits_{x\to 1} f(x)$

d) $f(1)$ e) $\lim\limits_{x\to -\infty} f(x)$ f) $\lim\limits_{x\to \infty} f(x)$

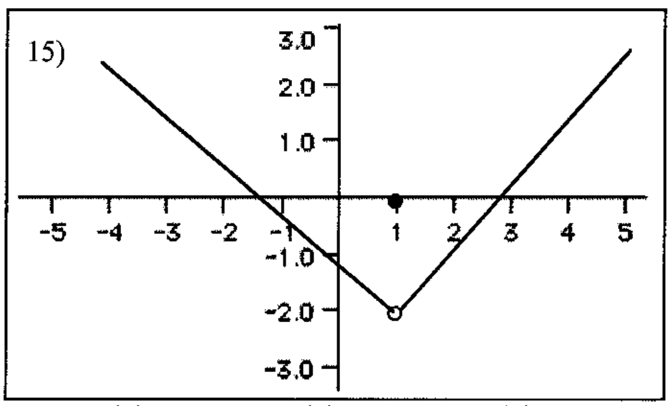
14)



a) $\lim\limits_{x\to 1^-} f(x)$ b) $\lim\limits_{x\to 1^+} f(x)$ c) $\lim\limits_{x\to 1} f(x)$

d) $f(1)$ e) $\lim\limits_{x\to -\infty} f(x)$ f) $\lim\limits_{x\to \infty} f(x)$

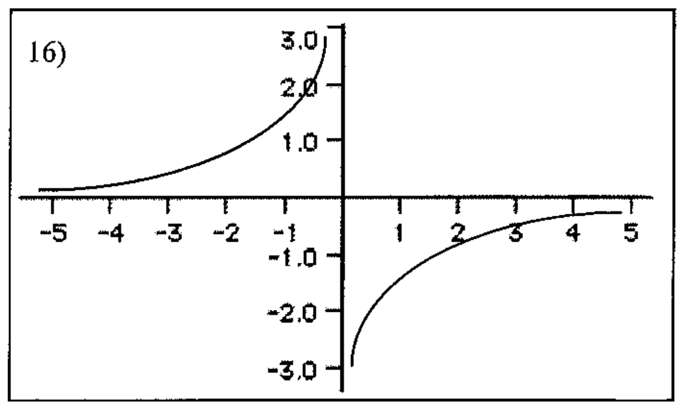
15)



a) $\lim\limits_{x\to 1^-} f(x)$ b) $\lim\limits_{x\to 1^+} f(x)$ c) $\lim\limits_{x\to 1} f(x)$

d) $f(1)$ e) $\lim\limits_{x\to -\infty} f(x)$ f) $\lim\limits_{x\to \infty} f(x)$

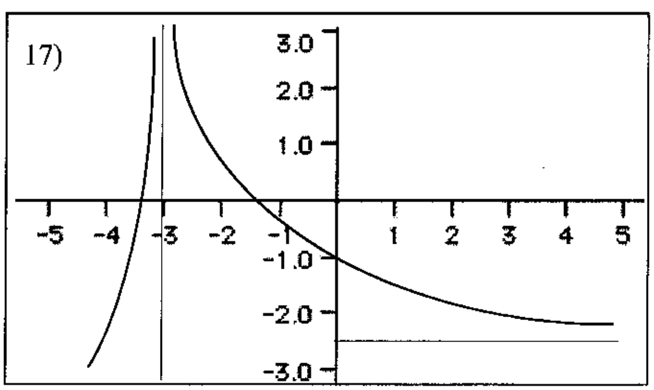
16)



a) $\lim\limits_{x\to 0^-} f(x)$ b) $\lim\limits_{x\to 0^+} f(x)$ c) $\lim\limits_{x\to 0} f(x)$

d) $f(0)$ e) $\lim\limits_{x\to -\infty} f(x)$ f) $\lim\limits_{x\to \infty} f(x)$

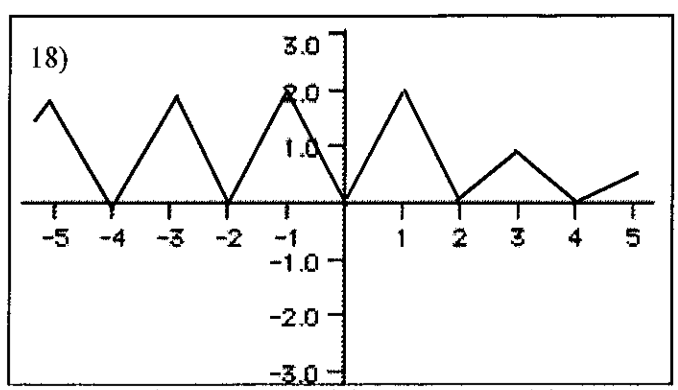
17)



a) $\lim\limits_{x\to -3^-} f(x)$ b) $\lim\limits_{x\to -3^+} f(x)$ c) $\lim\limits_{x\to -3} f(x)$

d) $f(-3)$ e) $\lim\limits_{x\to -\infty} f(x)$ f) $\lim\limits_{x\to \infty} f(x)$

18)



a) $\lim\limits_{x\to 0^-} f(x)$ b) $\lim\limits_{x\to 0^+} f(x)$ c) $\lim\limits_{x\to 0} f(x)$

d) $f(0)$ e) $\lim\limits_{x\to -\infty} f(x)$ f) $\lim\limits_{x\to \infty} f(x)$

Limit Worksheet

Consider the chart below.

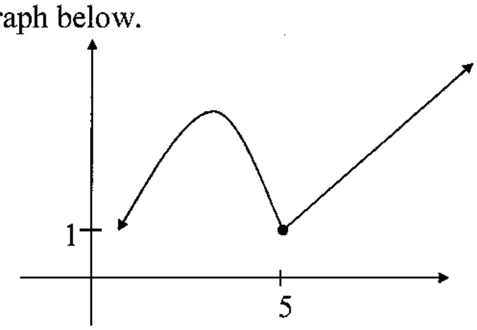
x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	3.1	3.01	3.001	3	4.001	4.01	4.1

1. Find $f(2)$.

2. Find $\lim_{x \to 2} f(x)$. Explain your answer.

3. Sketch a graph of the function around the value $x = 2$.

Consider the graph below.



1. Approximate the values for the chart below.

x	4.9	4.99	4.999	5	5.001	5.01	5.1
$f(x)$							

2. Find $\lim_{x \to 5} f(x)$. Explain your answer.

3. Find $f(5)$.

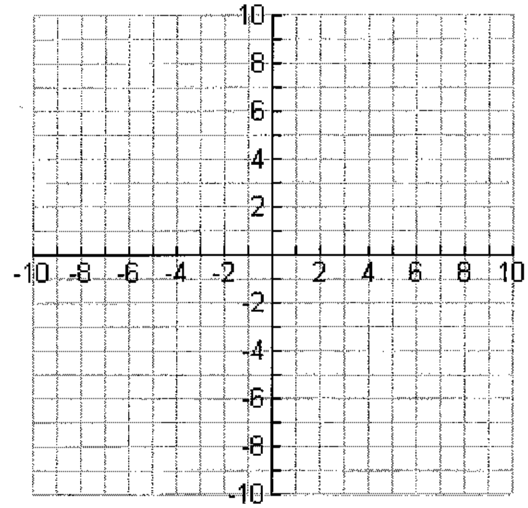
1. Sketch a graph that has the following properties.

$f(3) = 2$

$\lim_{x \to 3^-} f(x) = 4$

$\lim_{x \to 3^+} f(x) = 0$

$f(x) > 0$ for all values of x



2. Refer to the graph of $g(x)$ shown below in order to answer the following questions. If a limit does not exist explain why.

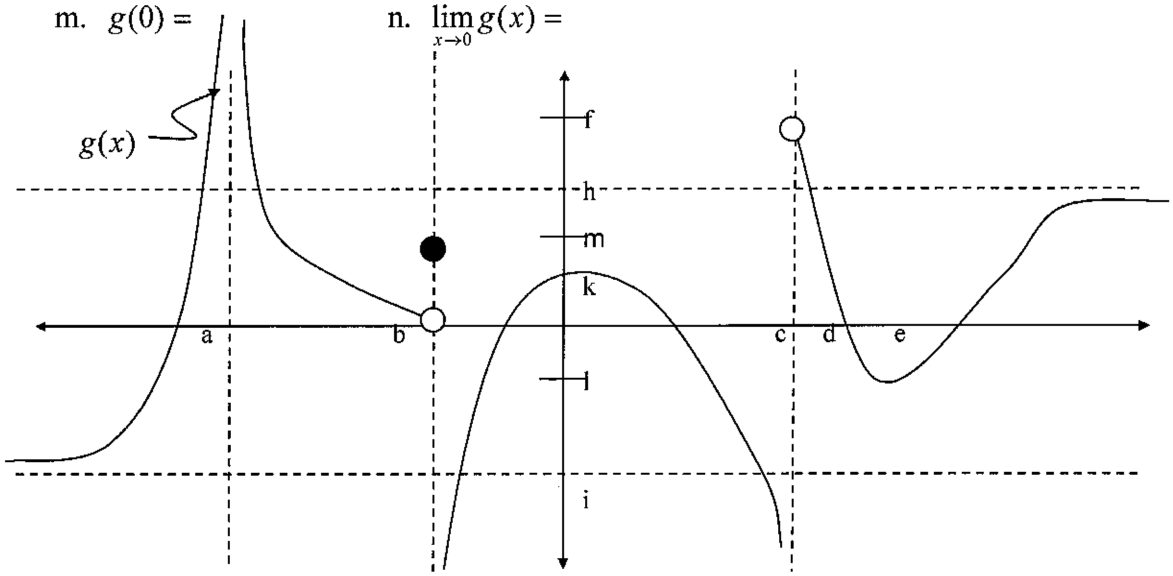
a. $\lim_{x \to \infty} g(x) =$ b. $\lim_{x \to -\infty} g(x) =$ c. $\lim_{x \to a^+} g(x) =$

d. $\lim_{x \to a^-} g(x) =$ e. $\lim_{x \to a} g(x) =$ f. $\lim_{x \to b^+} g(x) =$

g. $\lim_{x \to b^-} g(x) =$ h. $\lim_{x \to b} g(x) =$ i. $\lim_{x \to c} g(x) =$

j. $\lim_{x \to d} g(X) =$ k. $g(a) =$ l. $g(b) =$

m. $g(0) =$ n. $\lim_{x \to 0} g(x) =$



Evaluate the following limits.

1. $\lim_{x \to 2} \frac{3x+4}{x+1}$

2. $\lim_{x \to 0^+} \sqrt{x}$

3. $\lim_{x \to 0^-} \sqrt{x}$

4. $\lim_{x \to 0} \sqrt{x}$

5. $\lim_{x \to 0} \frac{\sin x}{x}$

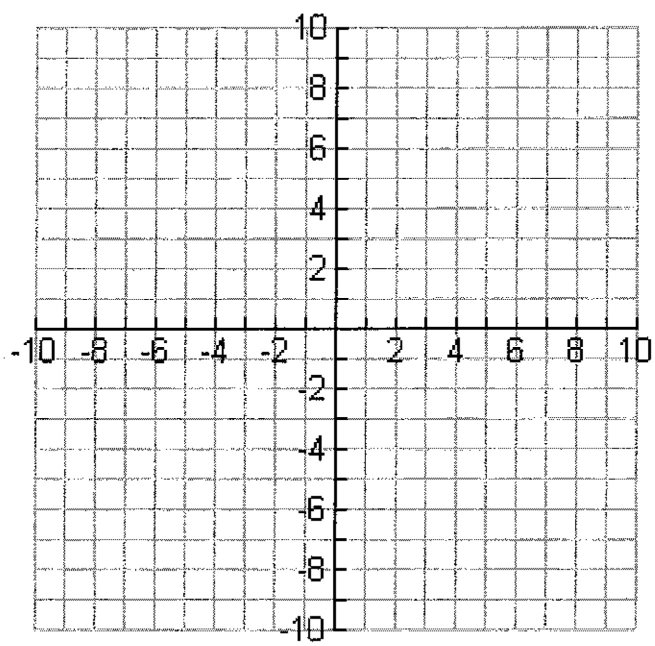
6. $\lim_{x \to \pi} \frac{\sin(x-\pi)}{x-\pi}$

7. $\lim_{h \to 0} \frac{2(3+h)-2(3)}{h}$

8. $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

1. Sketch a graph of a function with the following properties:

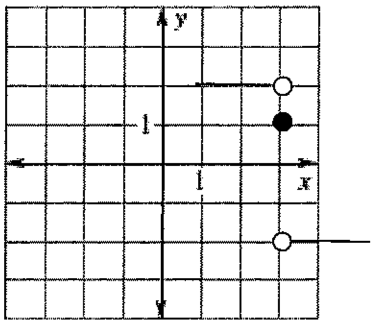
- $\lim_{x \to 0^+} f(x) = 2$
- $\lim_{x \to 3} f(x) = 0$
- $f(0) = 0$
- $\lim_{x \to 0^-} f(x) = -1$
- $\lim_{x \to -1} f(x) = 4$
- $f(3) = 1$



Honors Pre-Calculus **Limits Worksheet #5** **Name________________________**
May 2014

Use the graph to estimate the limits and function values, or explain why the limits do not exist or the function values are undefined.

1.



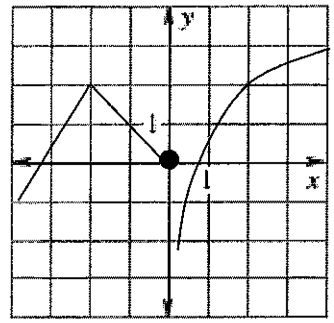
a. $\lim_{x \to 3^-} =$ ____________

b. $\lim_{x \to 3^+} =$ ____________

c. $\lim_{x \to 3} =$ ____________

d. $f(3) =$ ____________

2.



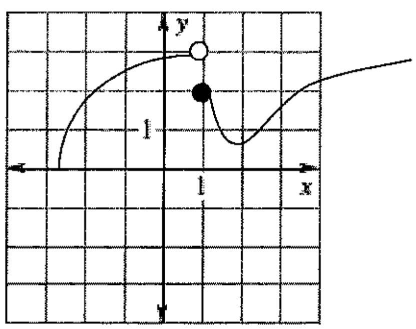
a. $\lim_{x \to -2^-} =$ ____________

b. $\lim_{x \to -2^+} =$ ____________

c. $\lim_{x \to -2} =$ ____________

d. $f(-2) =$ ____________

3.



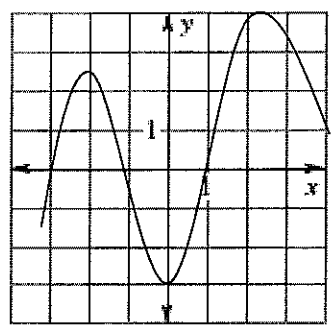
a. $\lim_{x \to 1^-} =$ ____________

b. $\lim_{x \to 1^+} =$ ____________

c. $\lim_{x \to 1} =$ ____________

d. $f(1) =$ ____________

4.



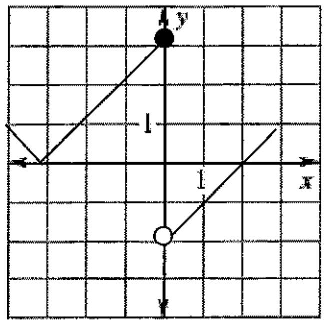
b. $\lim_{x \to 0^-} =$ ____________

c. $\lim_{x \to 0^+} =$ ____________

d. $\lim_{x \to 0} =$ ____________

e. $f(0) =$ ____________

5.



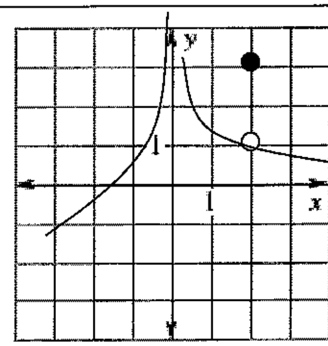
a. $\lim_{x\to 0^-} =$ ____________

b. $\lim_{x\to 0^+} =$ ____________

c. $\lim_{x\to 0} =$ ____________

d. $f(0) =$ ____________

6.



a. $\lim_{x\to 2^-} =$ ____________

b. $\lim_{x\to 2^+} =$ ____________

c. $\lim_{x\to 2} =$ ____________

d. $f(2) =$ ____________

Determine the limit.

7. $\lim_{x\to -\frac{1}{2}} 3x^2(2x-1)$

8. $\lim_{x\to -4} (x+3)^{1997}$

9. $\lim_{x\to -3} \frac{x^2+4x+3}{x^2-3}$

10. $\lim_{x\to 0} e^x \cos x$

11. $\lim_{x\to -2} \sqrt{x-2}$

12. $\lim_{x\to 0} \frac{1}{x^2}$

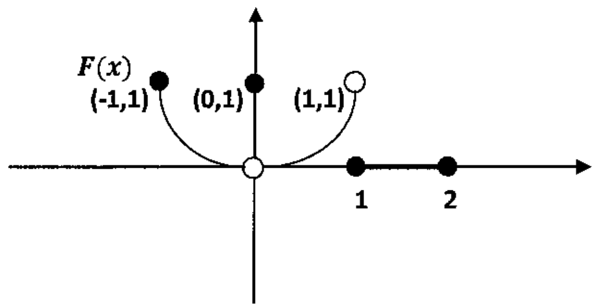
13. $\lim_{x\to 1} \frac{x-1}{x^2-1}$

14. $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4}$

15. $\lim_{x\to 0} \frac{(2+x)^3-8}{x}$

16. $\lim_{x\to 0} \frac{\sin 2x}{x}$ *hint: graph this one!

17.



a. $\lim_{x\to 0^-} f(x) =$ ___________

b. $\lim_{x\to 0^+} f(x) =$ ___________

c. $\lim_{x\to 0} f(x) =$ ___________

d. $\lim_{x\to 1^-} f(x) =$ ___________

e. $\lim_{x\to 1^+} f(x) =$ ___________

f. $\lim_{x\to 1} f(x) =$ ___________

g. $\lim_{x\to 2^-} f(x) =$ ___________

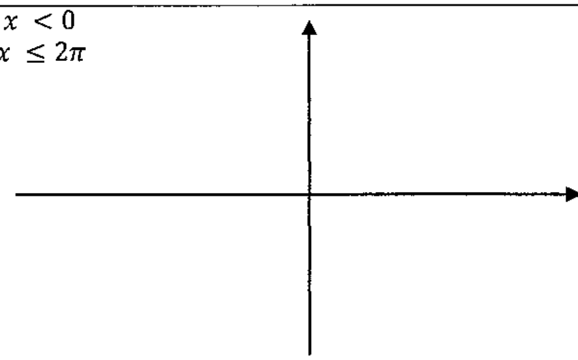
h. $\lim_{x\to 2^+} f(x) =$ ___________

i. $\lim_{x\to 2} f(x) =$ ___________

j. $f(0) =$ ___________

k. On the interval $[-1,1]$, f(x) is discontinuous at x = ___________

18. Given the piecewise function $f(x) = \begin{cases} \sin x, & -2\pi \le x < 0 \\ \cos x, & 0 \le x \le 2\pi \end{cases}$



a. Draw the graph

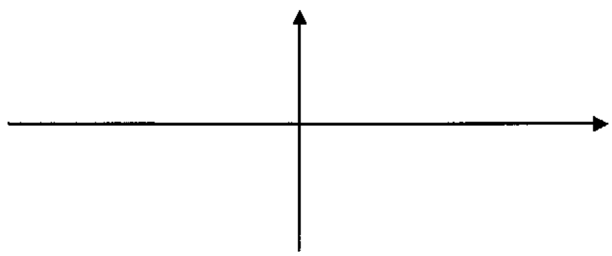
b. At what points does ***<u>only</u>*** the left hand limit exist?

c. At what point does ***<u>only</u>*** the right hand limit exist?

19. $\lim_{x \to 0} \dfrac{\frac{1}{x+2} - \frac{1}{2}}{x}$	**20.** $\lim_{x \to \infty} \; 2\cos\left(\frac{1}{x}\right) + 1$
21. $\lim_{x \to \infty} \dfrac{e^{-x}}{x}$	**22.** $\lim_{x \to 0^+} \; \csc x$

23. Sketch a possible graph for a function $f(x)$ that has the stated properties.

$f(4)$ exists (is defined), $\lim_{x \to 4} f(x)$ exists, but $f(x)$ is **not** continuous at $x = 4$



Answers:

1a. 2	**1b.** -2	**1c.** does not exist	**1d.** 1
2a. 2	**2b.** 2	**2c.** 2	**2d.** 2
3a. 3	**3b.** 2	**3c.** does not exist	**3d.** 2
4a. -3	**4b.** -3	**4c.** -3	**4d.** -3
5a. 3	**5b.** -2	**5c.** does not exist	**5d.** 3
6a. 1	**6b.** 1	**6c.** 1	**6d.** 3
7. $-\frac{3}{2}$	**8.** -1	**9.** 0	**10.** 1
11. does not exist	**12.** does not exist	**13.** $\frac{1}{2}$	**14.** $\frac{1}{4}$
15. 12	**16.** 2	**17a.** 0	**17b.** 0
17c. 0	**17d.** 1	**17e.** 0	**17f.** does not exist
17g. 0	**17h.** does not exist	**17i.** does not exist	**17j.** 1
17k. $x = 0, 1$	**18b.** 2π	**18c.** -2π	**19.** $-\frac{1}{4}$
20. 3	**21.** 0	**22.** ∞	**23.** answers will vary

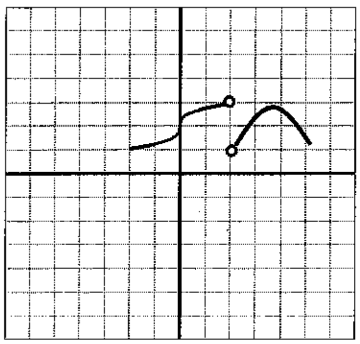
CALCULUS Name: ______________________

WORKSHEET L.1-1

Refer to the graph to find each limit, if it exists:

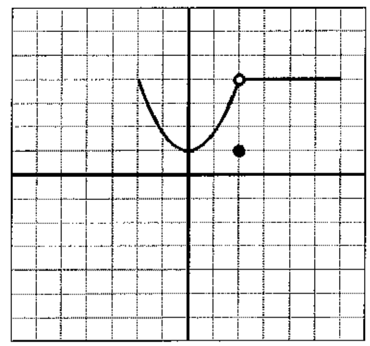
a. $\lim_{x \to 2^-} f(x)$ **b.** $\lim_{x \to 2^+} f(x)$ **c.** $\lim_{x \to 2} f(x)$ **d.** $\lim_{x \to 0^-} f(x)$ **e.** $\lim_{x \to 0^+} f(x)$ **f.** $\lim_{x \to 0} f(x)$

1.



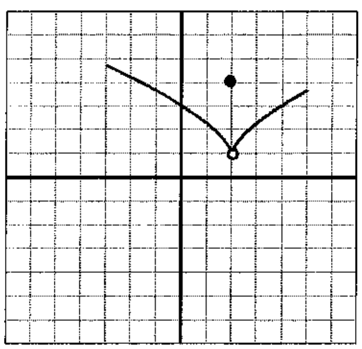
a. _____ **b.** _____ **c.** _____
d. _____ **e.** _____ **f.** _____

2.



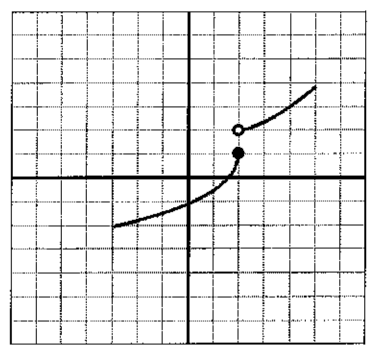
a. _____ **b.** _____ **c.** _____
d. _____ **e.** _____ **f.** _____

3.



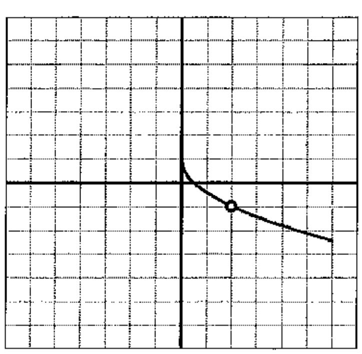
a. _____ **b.** _____ **c.** _____
d. _____ **e.** _____ **f.** _____

4.



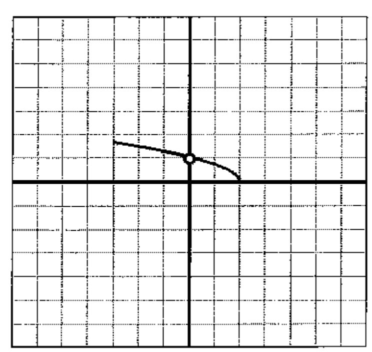
a. _____ **b.** _____ **c.** _____
d. _____ **e.** _____ **f.** _____

5.



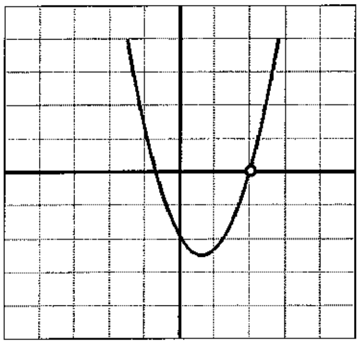
a. _____ **b.** _____ **c.** _____
d. _____ **e.** _____ **f.** _____

6.



a. _____ **b.** _____ **c.** _____
d. _____ **e.** _____ **f.** _____

7.

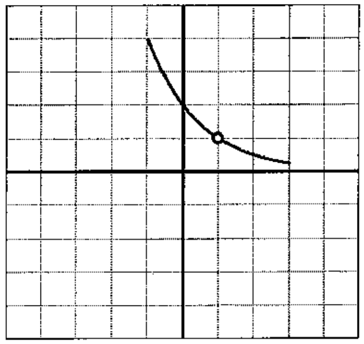


a. $\lim_{x\to 2} f(x) =$ _____

b. $\lim_{x\to 0} f(x) =$ _____

c. $f(2) =$ _____

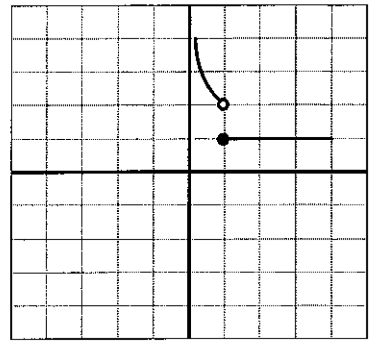
8.



a. $\lim_{x\to 1} f(x) =$ _____

b. $\lim_{x\to 0} f(x) =$ _____

9.



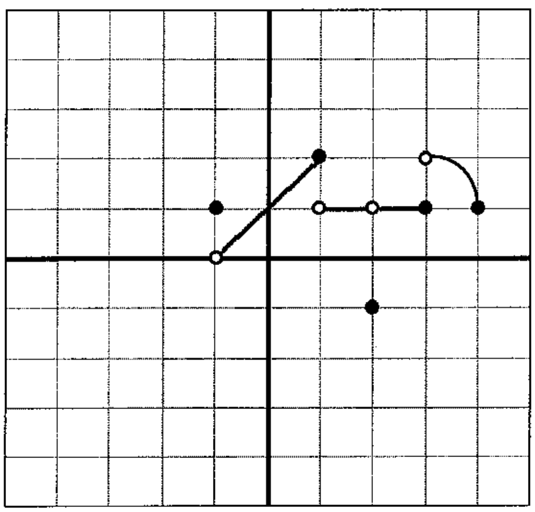
a. $\lim_{x\to 1^-} f(x) =$ _____

b. $\lim_{x\to 1^+} f(x) =$ _____

c. $\lim_{x\to 1} f(x) =$ _____

d. $f(1) =$ _____

10.



True or false?

_____ **a.** $\lim_{x\to 2} f(x) = -1$

_____ **b.** $\lim_{x\to -1^+} f(x) = 1$

_____ **c.** $\lim_{x\to 1^+} f(x) = 1$

_____ **d.** $\lim_{x\to 2} f(x)$ exists

_____ **e.** $\lim_{x\to 3} f(x) = 1$

_____ **f.** $\lim_{x\to 1} f(x)$ DNE

_____ **g.** $\lim_{x\to 3^-} f(x) = 1$

_____ **h.** $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

_____ **i.** $\lim_{x\to 0} f(x)$ exists

_____ **j.** $\lim_{x\to 2} f(x) = 1$

_____ **k.** $\lim_{x\to c} f(x)$ exists at every c on the interval (-1,1)

_____ **l.** $\lim_{x\to c} f(x)$ exists at every c on the interval (1,3)

Warm-Up

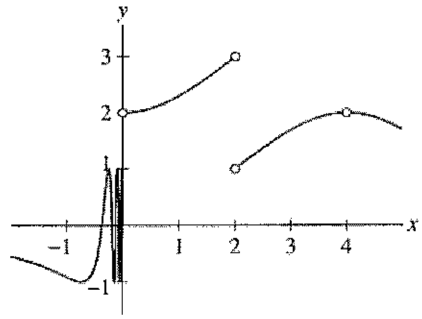


FIGURE 7

Evaluate the following. If a limit DNE, explain why.

1. $\lim_{x\to 2} f(x)$
2. $f(2) =$
3. $\lim_{x\to 4} f(x)$
4. $f(4) =$
5. $\lim_{x\to 0} f(x)$

Properties of Limits

1. $\lim_{x\to c} a = a$
2. $\lim_{x\to c} x = c$
3. $\lim_{x\to c} x^n = c^n$
4. $\lim_{x\to c} ax = ac$

THEOREM 1 Basic Limit Laws Assume that $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist. Then:

(i) Sum Law:

$$\lim_{x\to c}\left(f(x)+g(x)\right)=\lim_{x\to c} f(x)+\lim_{x\to c} g(x)$$

(ii) Constant Multiple Law: For any number k,

$$\lim_{x\to c} kf(x)=k\lim_{x\to c} f(x)$$

(iii) Product Law:

$$\lim_{x\to c} f(x)g(x)=\left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c} g(x)\right)$$

(iv) Quotient Law: If $\lim_{x\to c} g(x)\neq 0$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

http://archives.math.utk.edu/visual.calculus/1/limits.18/index.html

Evaluate the following using the limit laws:

1. $\lim_{x\to -3} 3x+4$

2. $\lim_{x\to 2} x(x+1)(x+2)$

Given $\lim_{x \to c} f(x) = 5$ and $\lim_{x \to c} g(x) = -4$

Evaluate:

1. $\lim_{x \to c} [6g(x)] =$

2. $\lim_{x \to c} [3f(x) - 2g(x)] =$

3. $\lim_{x \to c} \left[\frac{f(x)}{g(x)}\right] =$

Today, we solve limits Algebraically:

*Always try Direct Substitution first
*If it doesn't work then try to simplify algebraically and/or evaluate the limit graphically and numerically.

Finding a limit by substitution

$\lim_{x \to 1} x^3 - 4x^2 + 5x - 3 =$

When substitution doesn't work, try Algebra

* Your new function is a "function that agrees at all but one point."

$$\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$$

Examples:

1. $\lim_{x \to 3} \frac{x^2 - 1}{x + 1}$

2. $\lim_{x \to \frac{3\pi}{2}} \sin x$

3. $\lim_{x \to -5} \frac{x^2-25}{x^3+125}$

Kuta Software - Infinite Calculus

Name________________________

Evaluating Limits

Date________________ Period____

Evaluate each limit.

1) $\lim_{x \to -1} 5$

2) $\lim_{x \to -\frac{5}{2}} (-x + 2)$

3) $\lim_{x \to 2} (x^3 - x^2 - 4)$

4) $\lim_{x \to 1} \left(-\frac{x^2}{2} + 2x + 4\right)$

5) $\lim_{x \to 3} -\sqrt{x + 3}$

6) $\lim_{x \to \frac{3}{2}} -\sqrt{2x + 4}$

7) $\lim_{x \to 1} -\frac{x - 4}{x^2 - 6x + 8}$

8) $\lim_{x \to \frac{3}{2}} \frac{-x - 3}{x^2 + x + 1}$

9) $\lim_{x \to \pi} \sin(x)$

10) $\lim_{x \to \frac{3\pi}{4}} 2\cos(x)$

Critical thinking questions:

11) Give an example of a limit that evaluates to 4.

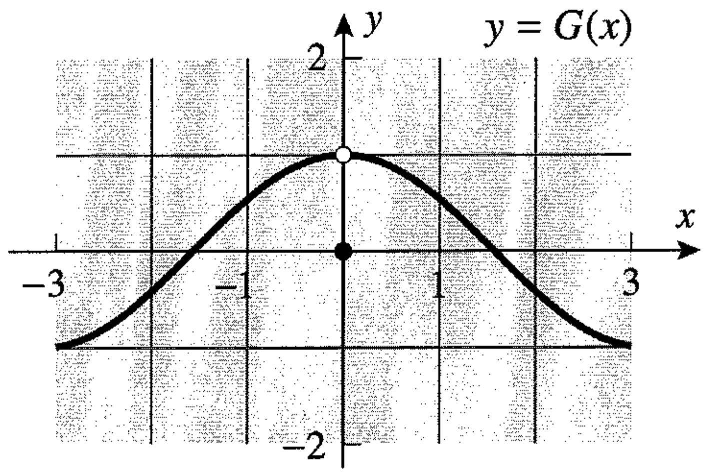
12) Give an example of a limit of a quadratic function where the limit evaluates to 9.

Limits Worksheet

1. For the function G graphed in the accompanying figure, find

(a) $\lim_{x \to 0^-} G(x)$ (b) $\lim_{x \to 0^+} G(x)$

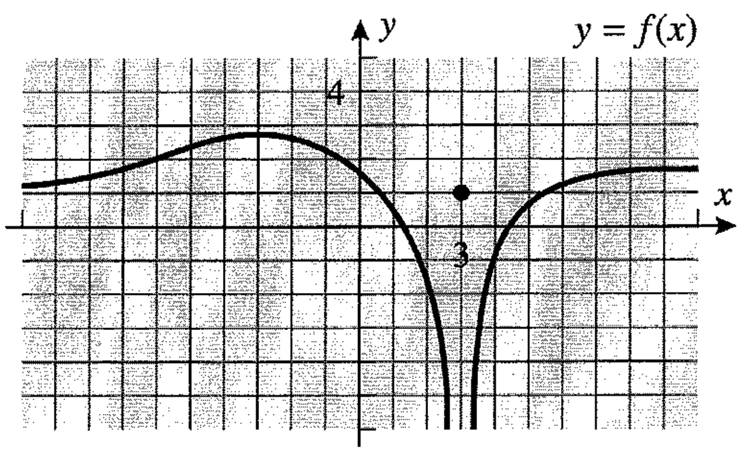
(c) $\lim_{x \to 0} G(x)$ (b) $G(0)$



2. For the function f graphed in the accompanying figure, find

(a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$

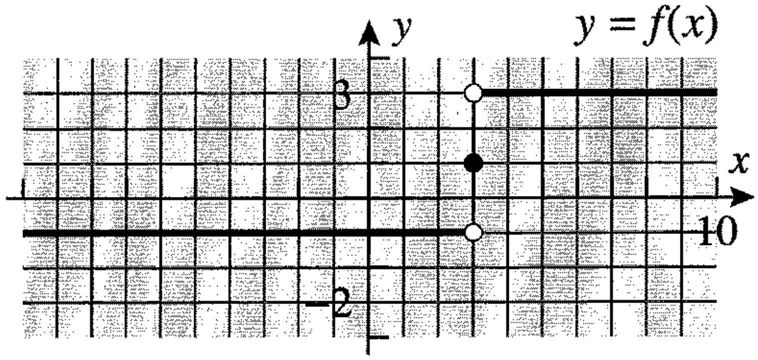
(c) $\lim_{x \to 3} f(x)$ (b) $f(3)$



3. For the function f graphed in the accompanying figure, find

(a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$

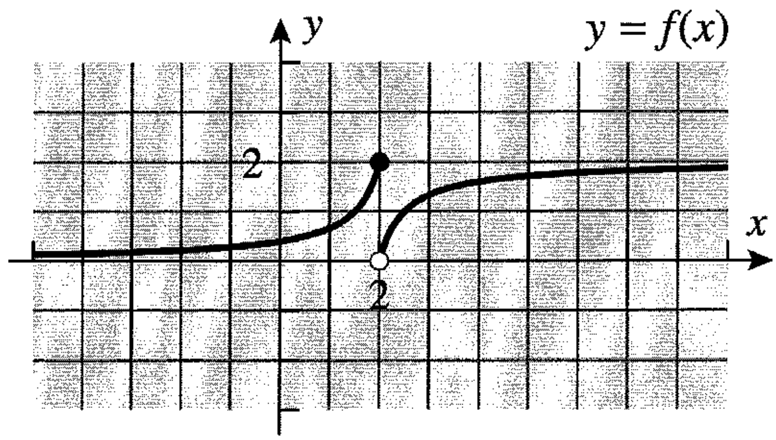
(c) $\lim_{x \to 3} f(x)$ (b) $f(3)$



4. For the function f graphed in the accompanying figure, find

(a) $\lim_{x \to -1^-} f(x)$ (b) $\lim_{x \to -1^+} f(x)$ (c) $\lim_{x \to -1} f(x)$

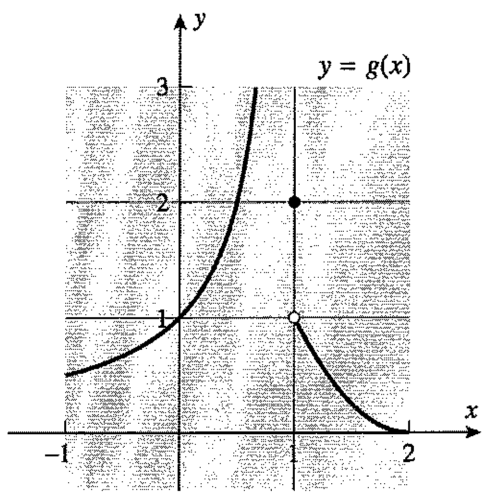
(c) $\lim_{x \to -1^-} f(x)$ (d) $\lim_{x \to -1^+} f(x)$ (e) $\lim_{x \to -1} f(x)$



5. For the function g graphed in the accompanying figure, find

(a) $\lim_{x\to 1^-} g(x)$ (b) $\lim_{x\to 1^+} g(x)$

(c) $\lim_{x\to 1} g(x)$ (b) $g(1)$

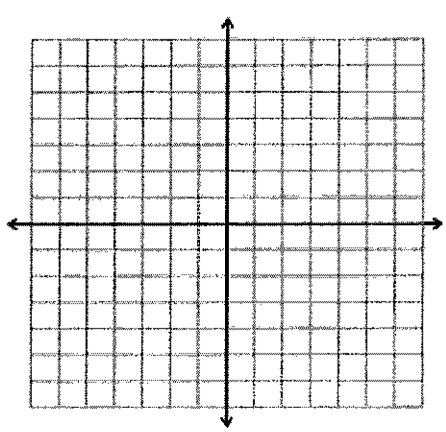


6. On the axes provided below, sketch a possible graph for a function f with the specified properties.

i. the domain is [-1,1]

ii. $f(-1) = f(0) = f(1) = 0$

iii. $\lim_{x\to -1^+} f(x) = \lim_{x\to 0} f(x) = \lim_{x\to 1^-} f(x) = 1$

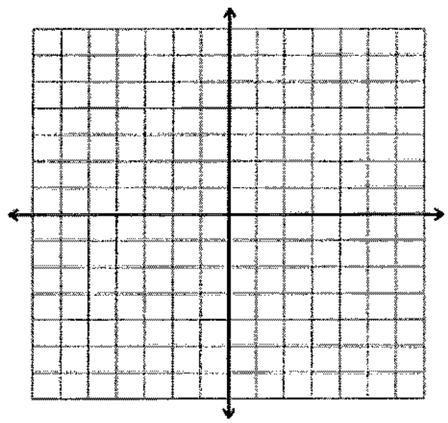


7. On the axes provided below, sketch a possible graph for a function f with the specified properties.

i. the domain is [-∞,1]

ii. $f(-2) = f(1) = 1$

iii. $\lim_{x \to -2} f(x) = +\infty$

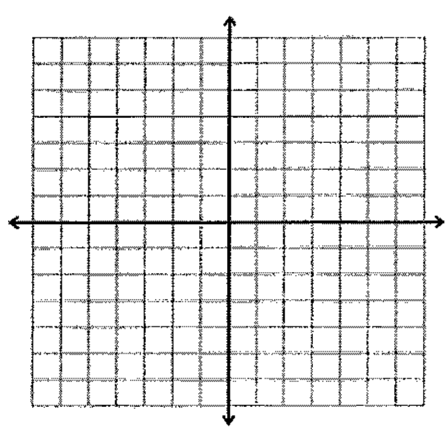


8. On the axes provided below, sketch a possible graph for a function f with the specified properties.

i. the domain is [-2,1]

ii. $f(-2) = f(0) = f(1) = 0$

iii. $\lim_{x \to -2^+} f(x) = 2$, $\lim_{x \to 0} f(x) = 0$, $\lim_{x \to 1^-} f(x) = 1$



Finding Limits Algebraically - Classwork

We are going to now determine limits without benefit of looking at a graph, that is $\lim\limits_{x \to a} f(x)$.

There are three steps to remember:
1) plug in a
2) Factor/cancel and go back to step 1
3) ∞, $-\infty$, or DNE

Example 1) find $\lim\limits_{x \to -2} x^2 - 4x + 1$

You can do this by plugging in.

Example 2) find $\lim\limits_{x \to -2} \dfrac{2x-6}{x-2}$

You can also do this by plugging in.

Example 3) find $\lim\limits_{x \to -2} \dfrac{x^2-2x-8}{x^2-4}$

Plug in and you get $\dfrac{0}{0}$ - no good

So attempt to factor and cancel

Example 4) find $\lim\limits_{x \to 1} \dfrac{x^2-2x+1}{x^3-1}$

Plug in and you get $\dfrac{0}{0}$ - no good

So attempt to factor and cancel

If steps 1 and 2 do not work (you have a zero in the denominator, your answer is one of the following:

∞	$-\infty$	**Does Not Exist (DNE)**

To determine which, you must split your limit into two separate limits.: $\lim\limits_{x \to a^-} f(x)$ and $\lim\limits_{x \to a+} f(x)$. Make a sign chart by plugging in a number close to a on the left side and determining its sign. You will also plug in a number close to a on the right side and determine its sign. **Each of these will be some form of ∞, either positive or negative.** Only if they are the same will the limit be ∞ or $-\infty$.

What this says is that in this case, $\lim\limits_{x \to a^-} f(x) =$ some form of ∞ and $\lim\limits_{x \to a+} f(x) =$ some form of ∞

You need to check whether they are the same.

Example 5) find $\lim\limits_{x \to 2} \dfrac{2x+5}{x-2}$

Step 1) Plug in - $\dfrac{9}{0}$ - no good Step 2) - No factoring/cancel So your answer is ∞, $-\infty$ or DNE

Example 6) find $\lim_{x\to 0}\frac{4}{x^2}$

Step 1) Plug in - $\frac{4}{0}$ - no good Step 2) - No factoring/cancel So your answer is ∞, $-\infty$ or DNE

Example 7) find $\lim_{x\to -3}\frac{x^2+2x-3}{x^2+6x+9}$

Example 8) find $\lim_{x\to 2}\frac{2x-4}{x^3-6x^2+12x-8}$

Example 9) $f(x)=\begin{cases}x^2-4, x\geq 1\\ -2x-1, x<1\end{cases}$ find $\lim_{x\to 1}f(x)$

Example 10) $f(x)=\begin{cases}\frac{x}{x-2}, x\geq 2\\ \frac{x-3}{x-2}, x<2\end{cases}$ find $\lim_{x\to 2}f(x)$

Example 11) $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

Finally, we are interested also in problems of the type: $\lim_{x\to\pm\infty}f(x)$. Here are the rules:

Write $f(x)$ as a fraction. 1) If the highest power of x appears in the denominator (bottom heavy), $\lim_{x\to\pm\infty}f(x)=0$

2) If the highest power of x appears in the numerator (top heavy), $\lim_{x\to\pm\infty}f(x)=\pm\infty$

plug in very large or small numbers and determine the sign of the answer

3) If the highest power of x appears both in the numerator and denominator

(powers equal), $\lim_{x\to\pm\infty}f(x)=\frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$

Example 12) $\lim_{x\to\infty}\frac{4x^2+50}{x^3-85}$

Example 13) $\lim_{x\to\infty}\frac{4x^3-5x^2+3x-1}{5x^3-7x-25}$

Example 14) $\lim_{x\to\infty}\frac{3x^3-23}{4x-1}$

Example 15) $\lim_{x\to-\infty}\frac{4x-5x^2+3}{\frac{1}{x}}$

Example 16) $\lim_{x\to\infty}\frac{\sqrt{x^2-3x}}{2x+1}$

Example 17) $\lim_{x\to-\infty}\frac{\sqrt{x^2-3x}}{2x+1}$

Finding Limits Algebraically - Homework

1) $\lim_{x \to 5} 12$

2) $\lim_{x \to 0} \pi$

3) $\lim_{x \to 2} 4x$

4) $\lim_{x \to 5} 3x^2 - 4x - 1$

5) $\lim_{x \to 0^-} 5x^3 - 7x^2 + 2^x - 2$

6) $\lim_{y \to -1} 3y^4 - 6y^3 - 2y$

7) $\lim_{x \to 4} \frac{2x-4}{x-1}$

8) $\lim_{x \to -2} \frac{x^2+4x+4}{x^2}$

9) $\lim_{x \to 1} \frac{2x-2}{x-1}$

10) $\lim_{x \to 4} \frac{x^2-16}{x-4}$

11) $\lim_{t \to -2} \frac{t^3+8}{t+2}$

12) $\lim_{x \to 2} \frac{x^2-4x+4}{x^2+x-6}$

13) $\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4}$

14) $\lim_{x \to 1} \frac{x^3+x^2-5x+3}{x^3-3x+2}$

15) $\lim_{x \to 3} \frac{x}{x-3}$

16) $\lim_{x \to 5} \frac{x}{x^2-25}$

17) $\lim_{y \to 6} \frac{y+6}{y^2-36}$

18) $\lim_{x \to 4} \frac{3-x}{x^2-2x-8}$

19) $\lim_{x \to 1} \frac{4}{x^2-2x+1}$

20) $\lim_{x \to 5} \frac{x}{|x-5|}$

21) $\lim_{x \to 3} \frac{-x^2}{x^2-6x+9}$

22) $f(x)=\begin{cases} x-1, x\geq 3 \\ 2x-3, x<3 \end{cases}$ find $\lim\limits_{x\to 3} f(x)$

23) $f(x)=\begin{cases} x^3-1, x\geq -1 \\ 2x, x<-1 \end{cases}$ find $\lim\limits_{x\to -1} f(x)$

24) $f(x)=\begin{cases} \dfrac{x-2}{x-1}, x\geq 1 \\ \dfrac{x}{x-1}, x<1 \end{cases}$ find $\lim\limits_{x\to 1} f(x)$

25) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$

26) Let $f(x)=\begin{cases} x^2-2x-3, x\neq 2 \\ k-3, x=2 \end{cases}$

find k such that $\lim\limits_{x\to 2} f(x)=f(2)$

27) $f(x)=\begin{cases} \dfrac{x^2-49}{x-7}, x\neq 7 \\ k^2-2, x=7 \end{cases}$

find k such that $\lim\limits_{x\to 7} f(x)=f(7)$

28) $\lim\limits_{x\to\infty} 6$

29) $\lim\limits_{x\to\infty}(-2x+11)$

30) $\lim\limits_{x\to-\infty}\left(3x^4-3x^3+5x^2+8x-3\right)$

31) $\lim\limits_{x\to\infty}\dfrac{2x-3}{4x+5}$

32) $\lim\limits_{x\to-\infty}\dfrac{7-3x^3}{2x^3+1}$

33) $\lim\limits_{x\to\infty}\dfrac{2}{5x-3}$

34) $\lim\limits_{x\to-\infty}\dfrac{2x+30}{6x^{12}-5}$

35) $\lim\limits_{x\to\infty}\dfrac{4x^4}{6x^3-19}$

36) $\lim\limits_{x\to-\infty}\dfrac{4x^2-3x-2-5x^3}{9x^2+9x+7}$

37) $\lim\limits_{x\to\infty}\dfrac{x}{\sqrt{x^2+4}}$

38) $\lim\limits_{x\to-\infty}\dfrac{x}{\sqrt{x^2+4}}$

39) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{3x^2+x}}{x^2-1}$

Limits BINGO

$-\frac{1}{9}$	4	3	5	1
7	$\frac{9}{2}$	0	2	∞
DNE not defined from left	$-\frac{3}{2}$	$-\infty$	-2	8
DNE	6	-4	$-\frac{1}{4}$	10
$\frac{11}{2}$	-3	-1	-5	DNE left lim ≠ right lim

Find each limit. Then locate your answer on the BINGO board and circle it (also write the problem number in the square). Work problems in any order until you have circled 5 answers in a row – horizontally, vertically, or diagonally. Submit this sheet and the work for a classwork grade. Answering all the problems correctly (and showing your work) will earn you 5 bonus points.

1. $\lim_{x\to 3}\frac{x^2-9}{x-3}$

2. $\lim_{x\to 3}\frac{x^2-x-6}{x-3}$

3. $\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}$

4. $\lim_{x\to 3}7$

5. $\lim_{x\to\frac{1}{2}}\frac{6x^2+5x-4}{2x-1}$

6. $\lim_{x\to -1}\left(\sqrt[3]{x}-\frac{2}{\sqrt[3]{x}}\right)$

7. $\lim_{x\to 2}\frac{x^2+4x-12}{x-2}$

8. $\lim_{x\to 2}\frac{x-2}{\frac{1}{x}-\frac{1}{2}}$

9. $\lim_{x\to\frac{1}{2}}\frac{5x+2}{2x}$

10. $\lim_{h\to 0}\frac{2-\sqrt{4+h}}{h}$

11. $\lim_{h\to 0}\frac{(5+h)^2-25}{h}$

12. $\lim_{x\to 4^-}\frac{|x-4|}{x-4}$

13. $\lim_{x\to 4}\frac{|x-4|}{x-4}$

14. $\lim_{x\to 4^+}\sqrt{x-4}-5$

15. $\lim_{x\to\infty}\frac{15x^2-2x+3}{5x^2-7}$

16. $\lim_{x\to 4^-}\sqrt{x-4}$

17. $\lim_{x\to\infty}\frac{\sqrt{4x^2+5}}{x-3}$

18. $\lim_{x\to-\infty}\frac{\sqrt{4x^2+5}}{x-3}$

19. $\lim_{x\to 4^+}\frac{1}{x-4}$

20. $\lim_{x\to 4^-}\frac{1}{x-4}$

21. $\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h}$

22. $\lim_{x\to-\infty}\frac{4-3x^3}{2x^3+3x-1}$

23. $\lim_{x\to\infty}\frac{10-3x}{(2x+1)^3}$

24. $\lim_{x\to\infty}\tan x$

25. $\lim_{x\to 1}\frac{x^2-5x+4}{x-1}$

Problems in **bold and underlined must** be worked algebraically.

__Brainstorm__: If I said a function was continuous what do you think I mean? Can you sketch a function that you think is continuous? Can you sketch one that is not?

A little more practice on...

Limits with piecewise functions:

$$f(x) = \begin{cases} x-2, & x \leq 0 \\ x^2+1, & x > 0 \end{cases}$$

Find $\lim_{x \to 0} f(x)$

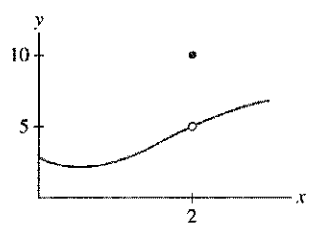
1.4 Continuity

Three Criteria for Continuity:

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Remember these 3 things!

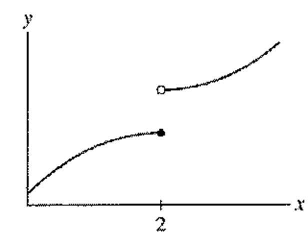
Example Determine which criteria are True.



$f(a)$ is defined

$\lim_{x \to a} f(x)$ exists

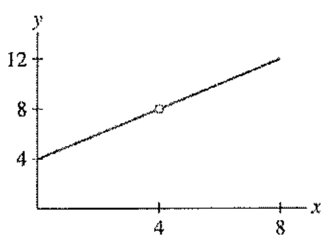
$\lim_{x \to a} f(x) = f(a)$



$f(a)$ is defined

$\lim_{x \to a} f(x)$ exists

$\lim_{x \to a} f(x) = f(a)$



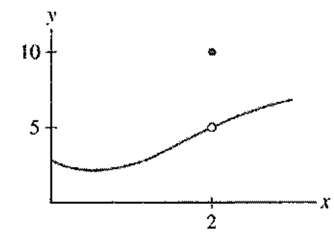
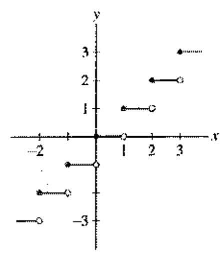
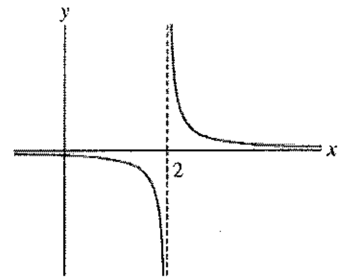
$f(a)$ is defined

$\lim_{x \to a} f(x)$ exists

$\lim_{x \to a} f(x) = f(a)$

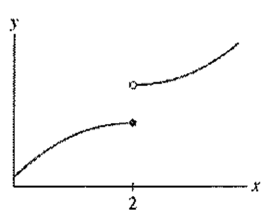
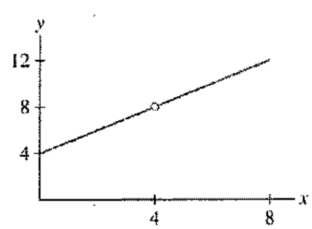
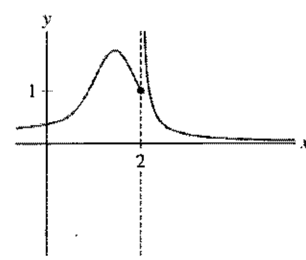
There are three *main* kinds of discontinuities:

1). hole/removable 2). jump 3). infinite

Jump and Infinite discontinuities are
<u>non-removable.</u>

<u>Example</u> State the type of discontinuity. If it's removable, find the value that will make the function continuous.

Is the function continuous? If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

$$f(x) = \frac{x^2 - 4}{x - 2}$$

State the intervals for which the function is continuous.

How can we define f(x) so that the function is continuous?

$$f(x) = \frac{x^2 - 4}{x - 2}, x \neq 2$$

Is the function continuous? If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

$$f(x) = \frac{1}{x+2}$$

State the interval for which the function is continuous.

Rewrite this function so that it is continuous.

$$f(x) = \begin{cases} 2x-1, & x < 1 \\ 5, & x = 1 \\ x^2, & x > 1 \end{cases}$$

$$f(x) = \begin{cases} 2x-1, & x < 1 \\ 5, & x = 1 \\ x^2, & x > 1 \end{cases}$$

What does *a* have to be in this function so that it is continuous?

$$f(x) = \begin{cases} 2x^2, & x < 2 \\ ax, & x \geq 2 \end{cases}$$

Is the function continuous? If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

$$f(x) = \frac{x}{x^2 - 9}$$

State the interval(s) for which the function is contiuous.

Is the function continuous? If not, state the x-value for which the function is not continuous. What kind of discontinuity is there?

$$y = [|\, x \,|]$$

Greatest Integer Function
The greatest integer less than or equal to x.

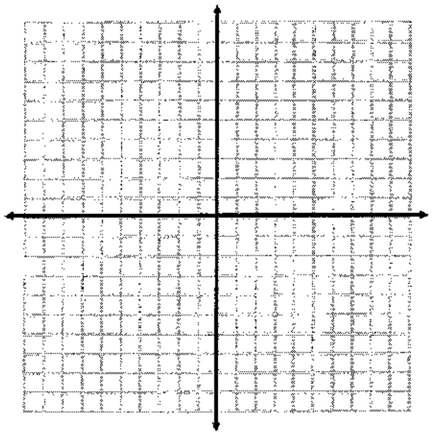
To put in your calculator
y1 = int (x)

* You can find "int" by pressing 2nd CATALOG (above the 0 key)

Honors GPS Calculus

Graph Sketching with Limits and Continuity

Spring 2016

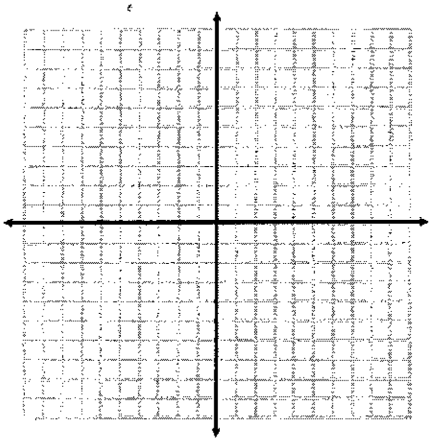


1) Sketch a graph with the following characteristics:

$\lim_{x\to 0^+} f(x) = 3$ $\lim_{x\to 0^-} f(x) = -5$ $f(0)$ is undefined

$\lim_{x\to 5} f(x) = \infty$ $\lim_{x\to -\infty} f(x) = 0$

Then identify any discontinuities, at what x-values they occur, and what type they are.

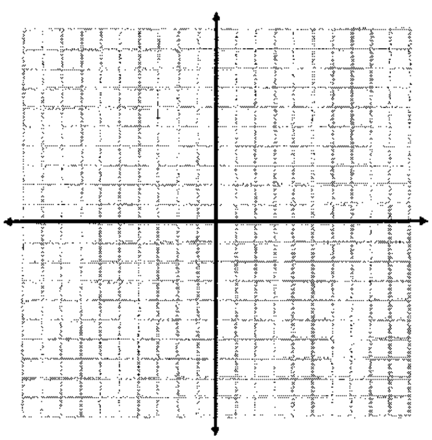


2) Sketch a graph with the following characteristics:

$\lim_{x\to -2^+} g(x) = -\infty$ $\lim_{x\to -2^-} g(x) = \infty$ $\lim_{x\to \pm\infty} g(x) = 2$

$\lim_{x\to 4} g(x) = 5$ $g(4) = -1$

Then identify any discontinuities, at what x-values they occur, and what type they are.



3) Sketch a graph with the following characteristics:

$\lim_{x\to 0^+} f(x) = -3$ $\lim_{x\to 0} f(x) = DNE$ $f(0) = 6$

$\lim_{x\to -4^+} f(x) = 8$ $\lim_{x\to -4^-} f(x) = 0$ $\lim_{x\to \infty} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = -3$

Then identify any discontinuities, at what x-values they occur, and what type they are.

Calculus Concepts
Limit Worksheet #1

Name________________________

Refer to the graph below to find each of the following. If the limit does not exist, explain why.

1. $\lim_{x \to +\infty} g(x) =$

2. $\lim_{x \to -\infty} g(x) =$

3. $\lim_{x \to a^+} g(x) =$

4. $\lim_{x \to a^-} g(x) =$

5. $\lim_{x \to a} g(x) =$

6. $\lim_{x \to 0} g(x) =$

7. $\lim_{x \to b^+} g(x) =$

8. $\lim_{x \to b^-} g(x) =$

9. $\lim_{x \to b} g(x) =$

10. $\lim_{x \to c} g(x) =$

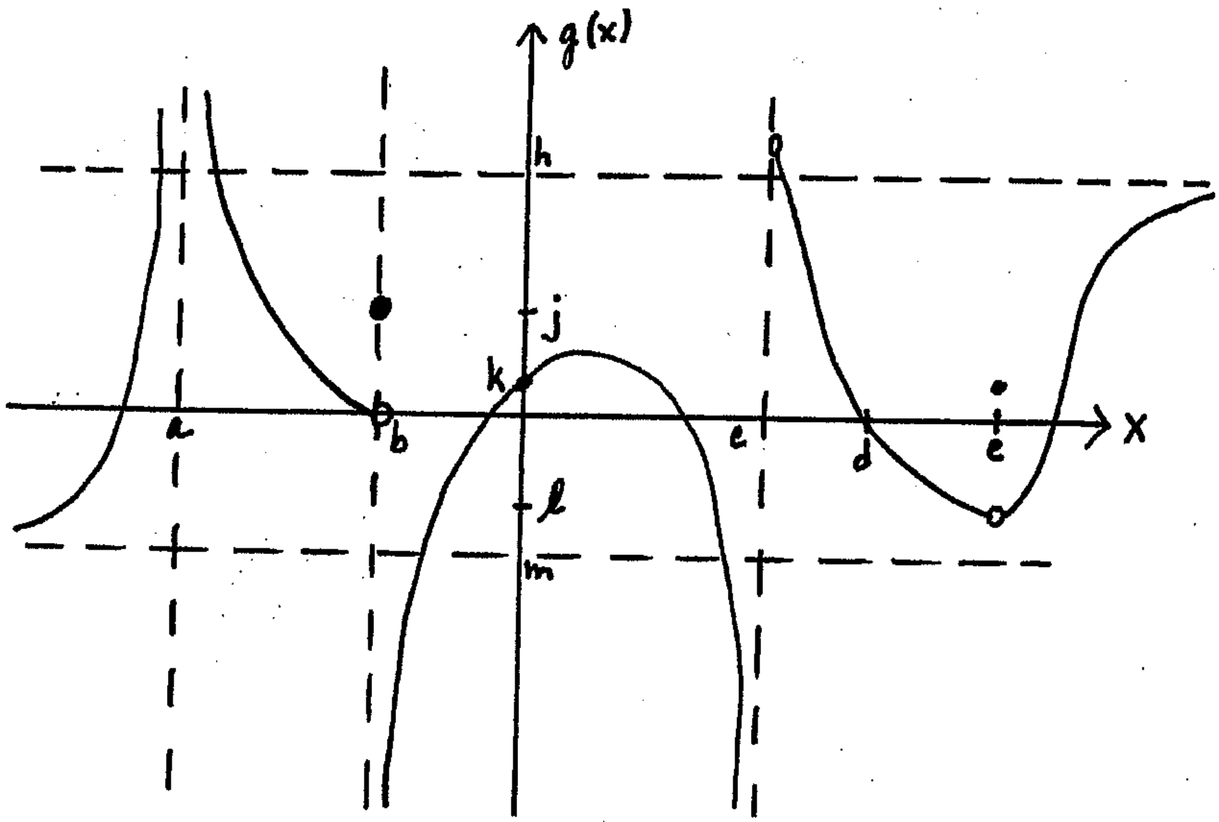
11. $\lim_{x \to d} g(x) =$

12. $\lim_{x \to e} g(x) =$

13. $g(e) =$

14. $g(0) =$

15. $g(b) =$

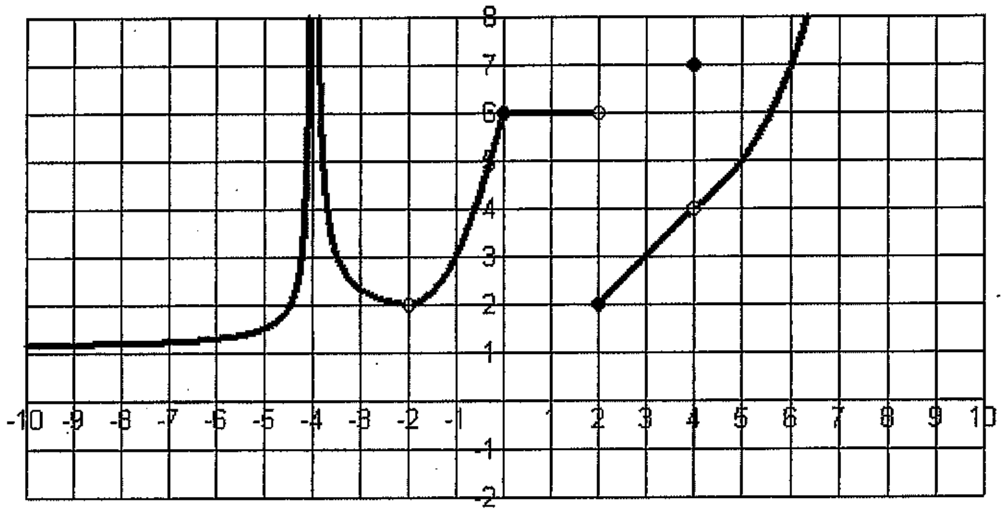


Limits and Continuity

Calculus Concepts
Unit 1

Name

Date



Use the graph of $f(x)$ shown above to answer questions 1-27.

1. $\lim_{x\to-\infty} f(x) =$ _____

2. $\lim_{x\to\infty} f(x) =$ _____

3. $\lim_{x\to-4^-} f(x) =$ _____

4. $\lim_{x\to-4^+} f(x) =$ _____

5. $\lim_{x\to-4} f(x) =$ _____

6. $f(-4) =$ _____

7. Is $f(x)$ continuous at $x=-4$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

8. $\lim_{x\to-2^-} f(x) =$ _____

9. $\lim_{x\to-2^+} f(x) =$ _____

10. $\lim_{x\to-2} f(x) =$ _____

11. $f(-2) =$ _____

12. Is $f(x)$ continuous at $x=-2$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

13. $\lim_{x \to 0^-} f(x) =$ ______

14. $\lim_{x \to 0^+} f(x) =$ ______

15. $\lim_{x \to 0} f(x) =$ ______

16. $f(0) =$ ______

17. Is continuous at $x = 0$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity

18. $\lim_{x \to 2^-} f(x) =$ ______

19. $\lim_{x \to 2^+} f(x) =$ ______

20. $\lim_{x \to 2} f(x) =$ ______

21. $f(2) =$ ______

22. Is continuous at $x = 2$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

23. $\lim_{x \to 4^-} f(x) =$ ______

24. $\lim_{x \to 4^+} f(x) =$ ______

25. $\lim_{x \to 4} f(x) =$ ______

26. $f(4) =$ ______

27. Is continuous at $x = 4$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

Limits and Continuity

Calculus Concepts
Unit 1 – Worksheet 3

Name

Date

1. Let $f(x)=\dfrac{x^2-9}{x+3}$.

   a. $\lim\limits_{x\to -3^-} f(x)=$ _____

   b. $\lim\limits_{x\to -3^+} f(x)=$ _____

   c. $\lim\limits_{x\to -3} f(x)=$ _____

   d. $f(-3)=$ _____

   e. Is continuous at $x=-3$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

2. Let $f(x)=\begin{cases} 3x+4 & x\le -2 \\ x^2+1 & x>-2 \end{cases}$.

   a. $\lim\limits_{x\to -2^-} f(x)=$ _____

   b. $\lim\limits_{x\to -2^+} f(x)=$ _____

   c. $\lim\limits_{x\to -2} f(x)=$ _____

   d. $f(-2)=$ _____

   e. Is continuous at $x=-2$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

3. Let $f(x)=\begin{cases} \dfrac{x^2+3x+2}{x+2} & x\ne -2 \\ \dfrac{1}{2} & x=-2 \end{cases}$.

   a. $\lim\limits_{x\to -2^-} f(x)=$ _____

   b. $\lim\limits_{x\to -2^+} f(x)=$ _____

   c. $\lim\limits_{x\to -2} f(x)=$ _____

   d. $f(-2)=$ _____

   e. Is continuous at $x=-2$? Why or why not? If it is not continuous, state the type of discontinuity and explain why it has that type of discontinuity.

4. Given $f(x)=\begin{cases}3x+2 & \text{if } x<4\\ 5x+k & \text{if } x\geq 4\end{cases}$.

Find the value of k such that $\lim_{x\to 4} f(x)$ exists.

5. Given $f(x)=\begin{cases}2x-a & \text{if } x<-3\\ ax+2b & \text{if } -3\leq x\leq 3\\ b-5x & \text{if } x>3\end{cases}$.

Find the values of a and b such that $\lim_{x\to -3} f(x)$ and $\lim_{x\to 3} f(x)$ both exist.

Limits and Continuity

Calculus Concepts
Unit 1 – Worksheet 4

Name

Date

Please show all work.
Full credit will not be given if appropriate work is not shown.

Find each of the following limits. Please use proper notation.

1. $\lim_{x\to 1}\left(x^2+2x-1\right)$

2. $\lim_{x\to 0}|x|$

3. $\lim_{x\to 3}\left(\dfrac{x-3}{x^2-9}\right)$

4. $\lim_{x\to 0}\left(\dfrac{1}{x}\right)$

5. $f(x)=\begin{cases}\dfrac{x^3-1}{x-1} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$; $\lim_{x\to 0} f(x)$

Use limits to find the constant a so that the function is continuous for all real values of x. Please focus on the use of proper notation.

6. $f(x)=\begin{cases}\dfrac{x^2-a^2}{x-a} & \text{if } x\neq a\\ 8 & \text{if } x=a\end{cases}$

Use limits to find the constant a so that the function is continuous for all real values of x. Please focus on the use of proper notation. Use the TI-Nspire to graph the function to confirm, visually, that the determined value of a makes the function continuous.

7. $f(x)=\begin{cases}3x+2 & \text{if } x<4\\ 5x+a & \text{if } x\geq 4\end{cases}$

Use limits to find the constants a and b so that the function is continuous for all real values of x. Please focus on the use of proper notation. Use the TI-Nspire to graph the function to confirm, visually, that the determined values of a and b make the function continuous.

8. $f(x)=\begin{cases}2 & \text{if } x\leq -1\\ ax+b & \text{if } -1<x<3\\ -2 & \text{if } x\geq 3\end{cases}$

Limits

Notation

$$\lim_{x \to c} f(x) = L$$

Read: The limit as x approaches c of f of x is equal to L.

Note: c is an x value, and L is a y value.

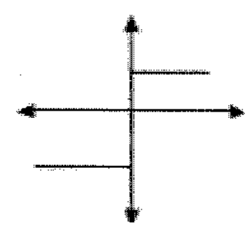
Analytical Approach

- Direct Substitution
- Factor and reduce
- Rationalization Technique
- The Squeeze Theorem
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$; $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

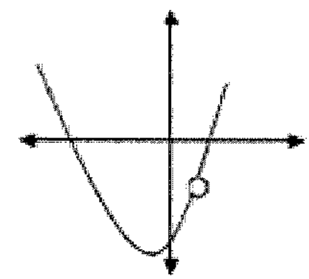
Numerical Approach

- Create a table of values.
- Pick x values that are close to c on either side of c.
- Make sure that the limit from the left equals the limit from the right.
- Your answer is the y value that f(x) approaches.

Graphical Approach



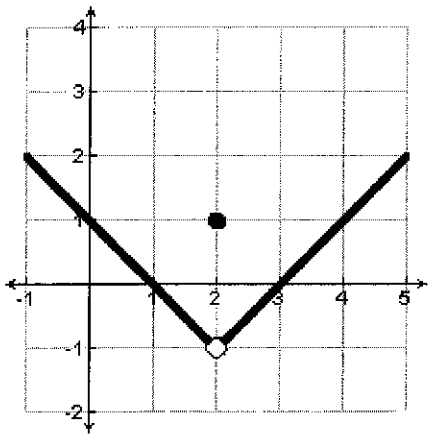
The limit at zero does not exist



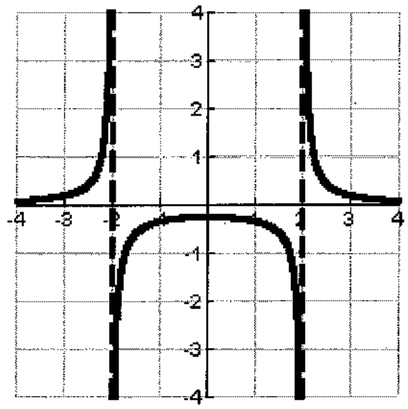
The limit is the y value of the function at the c value you are investigating. Make sure that the limit from the right equals the limit from the left.

Calculus Review Chapter 1

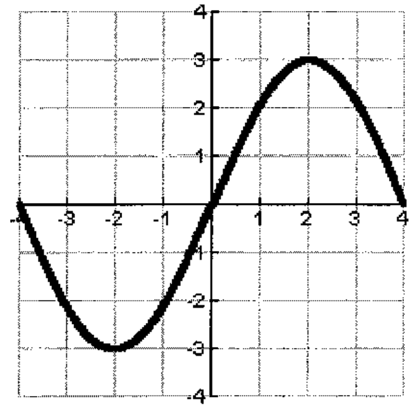
1\. Find $\lim_{x\to 2} f(x)$ for the graph.



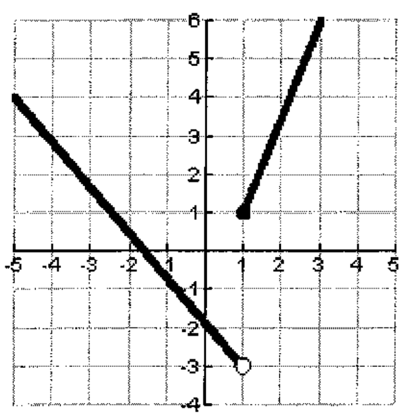
2\. Find $\lim_{x\to 2} f(x)$ for the graph.



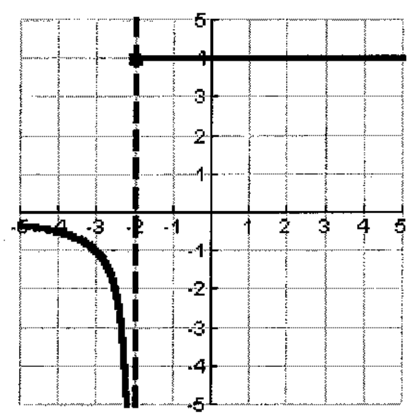
3\. Find $\lim_{x\to -2} f(x)$ for the graph.



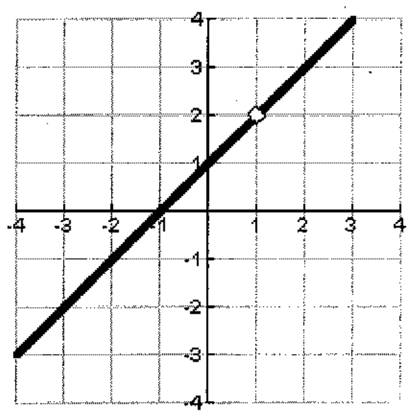
4\. Find $\lim_{x\to 1^+} f(x)$ for the graph.



5\. Find $\lim_{x\to -2^-} f(x)$ for the graph.



6\. Find $\lim_{x\to 1} f(x)$ for the graph.



7\. Which of the above graphs are discontinuous at a point? Name the point and give the reason the graph is not continuous there.

Find the limits algebraically (work must be shown).

8\. $\lim_{x\to 1}(2x-1)$

9\. $\lim_{x\to 2}\left(\dfrac{x-2}{x^2-2x}\right)$

10\. $\lim_{x\to 0}\left(\dfrac{\sin 3x}{x}\right)$

11\. $\lim_{x\to 1}\left(\dfrac{x^3-1}{x^2+2}\right)$

12\. $\lim_{x\to 1}\left(\dfrac{x^2+x-2}{x-3}\right)$

13\. $\lim_{x\to 2}\left(\sqrt{4x^2+9}\right)$

14\. $\lim_{x\to\infty}\left(\dfrac{2x^3+6x^2+5}{x^3+5}\right)$

15\. $\lim_{x\to\infty}\left(\dfrac{\sqrt{4x^2-1}}{x^2}\right)$

16\. $\lim_{x\to\infty}\left(\dfrac{x^2}{\sqrt{4x^2-1}}\right)$

17\. $\lim_{x\to 1} f(x),\ f(x)=\begin{cases}2x, & x\le 1\\ x+1, & x>1\end{cases}$

18\. $\lim_{x\to 0} f(x),\ f(x)=\begin{cases}x^2+1, & x<0\\ x-1, & x\ge 0\end{cases}$

60

If $\lim_{x\to 3} f(x) = -2$ and $\lim_{x\to 3} g(x) = 3$, find

19. $\lim_{x\to 3}(f(x)+g(x))$

20. $\lim_{x\to 3}(f(x)\cdot g(x))$

21. $\lim_{x\to 3}(3f(x)-2g(x))$

Find the limit any way you wish.

22. $\lim_{x\to -2}\left(\frac{x+2}{x^3+8}\right)$

23. $\lim_{x\to 0}\left(\frac{1-\cos^2 x}{x}\right)$

24. $\lim_{x\to 0}\left(\frac{\sqrt{x+4}-2}{x}\right)$

25. $\lim_{x\to 1^+}\left(\sqrt{x-1}\right)$

26. $\lim_{x\to 1^-}\left(\sqrt{x-1}\right)$

27. $\lim_{x\to 0}\left(\frac{|x|}{x}\right)$

Find all vertical and horizontal asymptotes.

28. $f(x)=\frac{7}{x-4}$

29. $f(x)=\frac{3x^2+2x-1}{x^2-1}$

30. $f(x)=\frac{x^2}{\sqrt{4x^2-1}}$

31. What is the relationship between the horizontal asymptote and the $\lim_{x\to\infty} f(x)$?

Find the point(s) at which the following have a removable discontinuity.

32. $f(x)=\frac{x+4}{x^2-16}$

33. $f(x)=\frac{x^2-16}{x+4}$

Where are the following functions not defined?

34. $f(x)=\frac{x^2+7x+12}{x^2+4x+3}$

35. $f(x)=\frac{x^2-1}{x+1}$

36. $f(x)=\frac{x^2+2x-3}{x^2+1}$

Where do the following functions not have a limit?

37. $f(x)=\frac{x^2+7x+12}{x^2+4x+3}$

38. $f(x)=\frac{x^2-1}{x+1}$

39. $f(x)=\frac{x^2+2x-3}{x^2+1}$

Where is each of the following not continuous? (It is possible that the function is continuous everywhere.)

40. $f(x)=\frac{x^2+7x+12}{x^2+4x+3}$

41. $f(x)=\frac{x^2-1}{x+1}$

42. $f(x)=\frac{x^2+2x-3}{x^2+1}$

Know the definition of continuity.